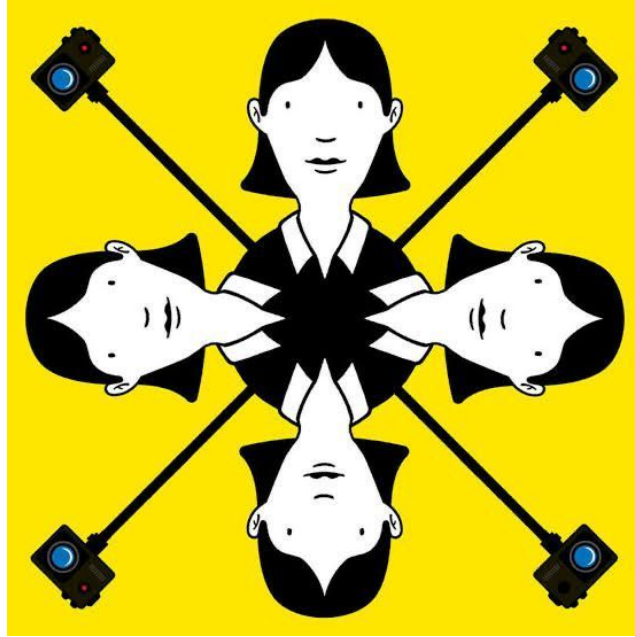


Simetrias no Plano

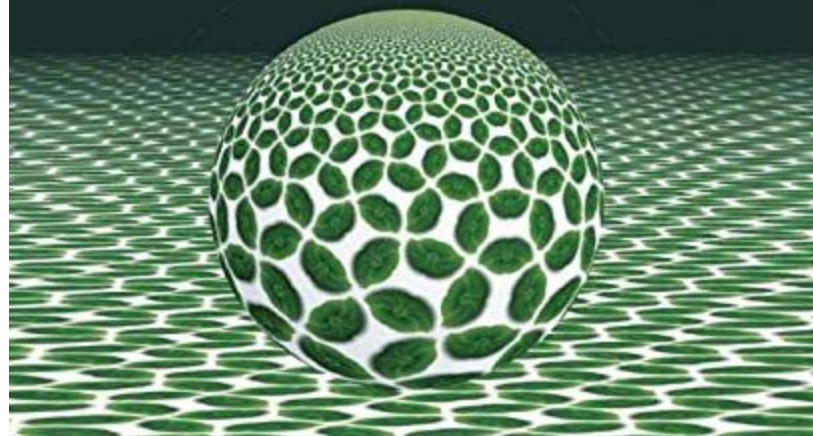


Lucas Rufino Martelotte

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The
SYMMETRIES
of
THINGS



John H. Conway • Heidi Burgiel • Chaim Goodman-Strauss

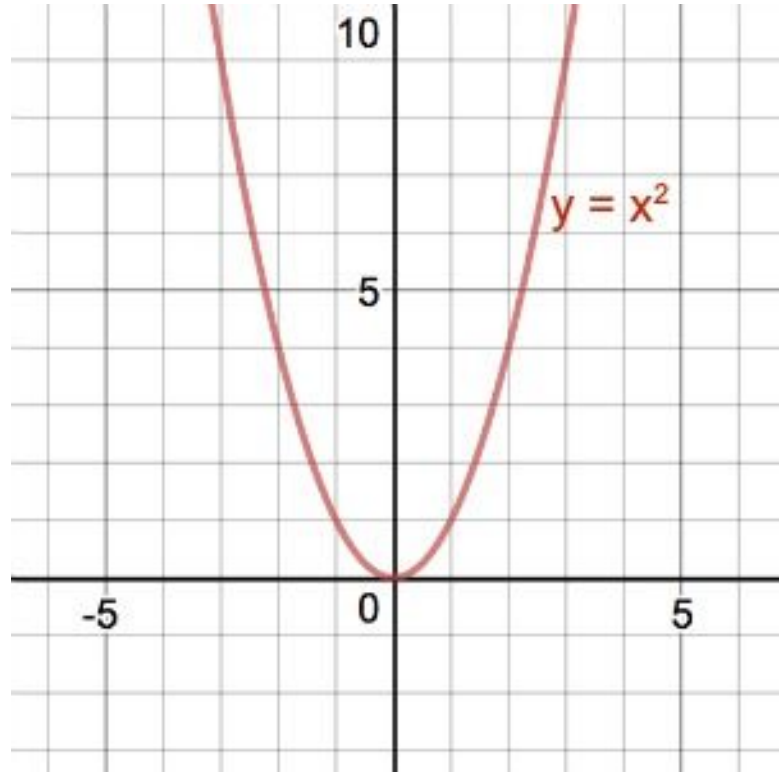
Copyrighted Material

O que é simetria?



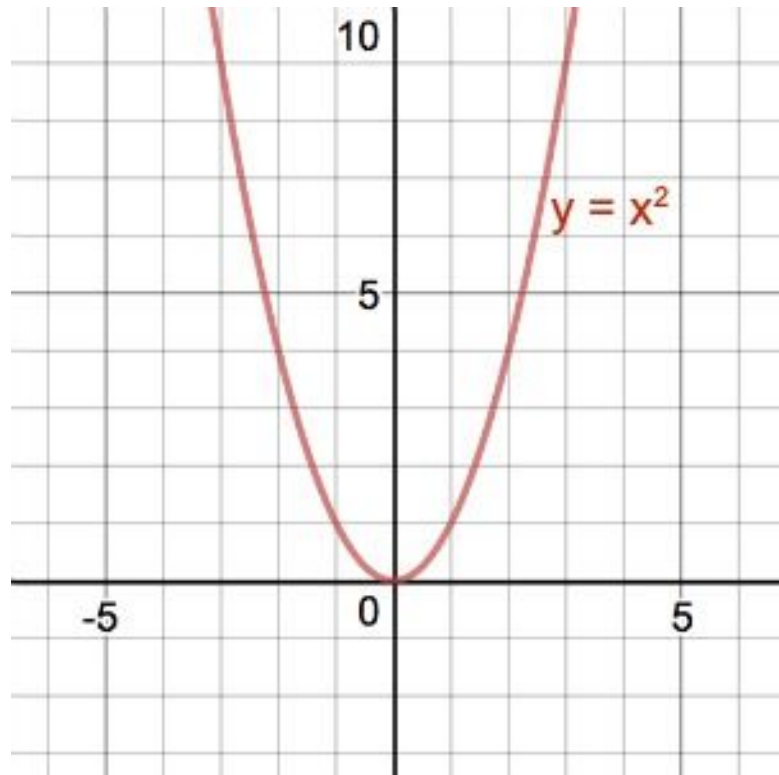


$$f(x) = x^2$$

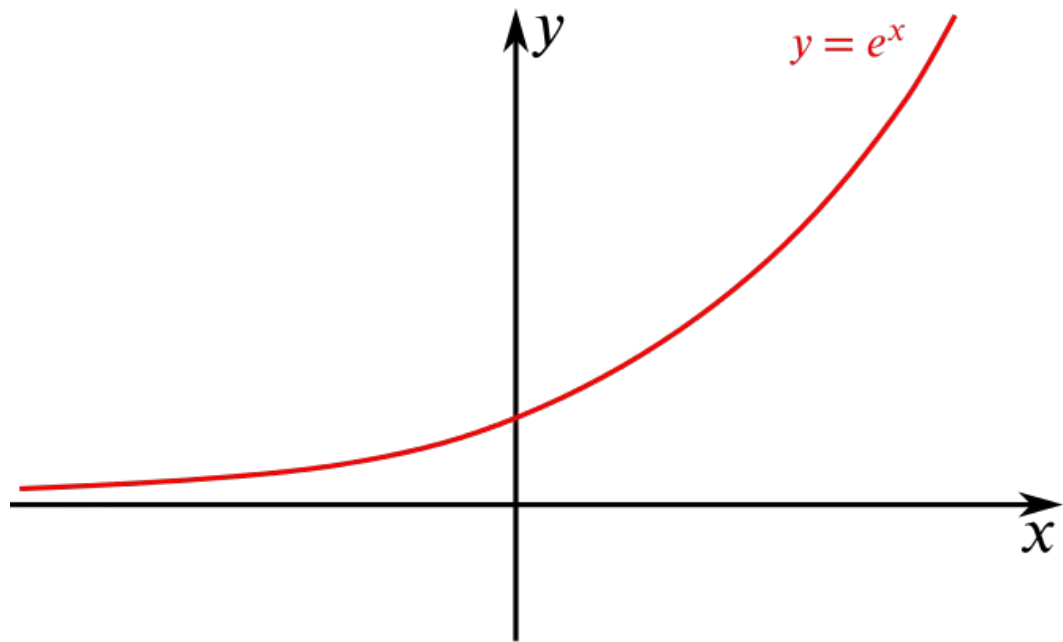


$$f(x) = x^2$$

$$f(-x) = f(x)$$

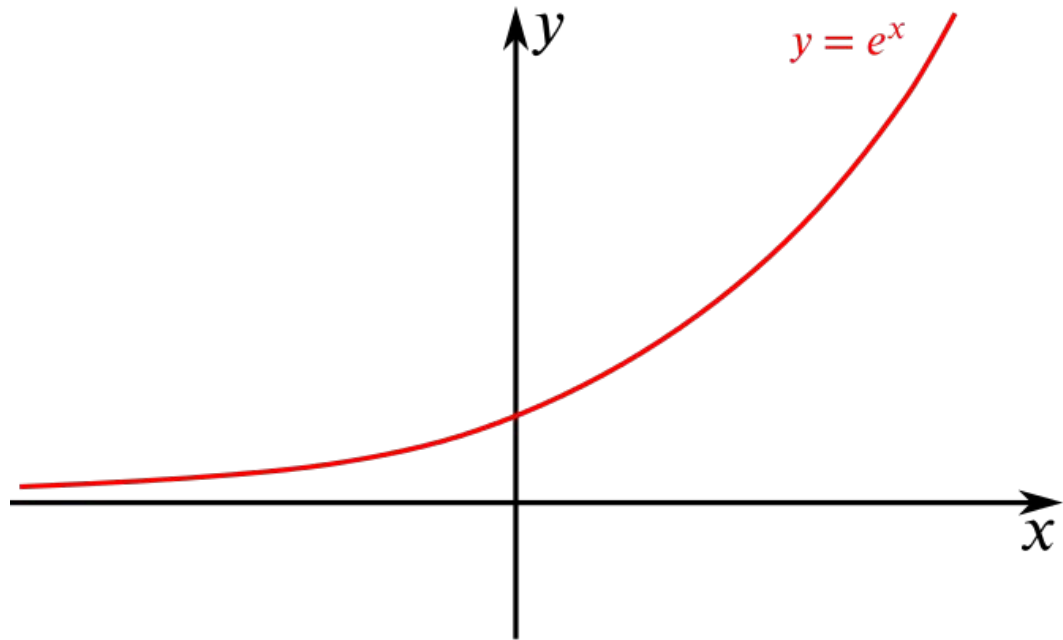


$$f(x) = e^x$$



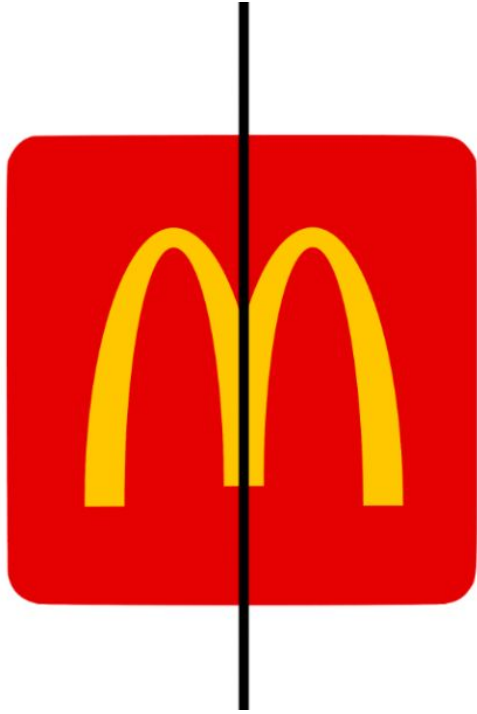
$$f(x) = e^x$$

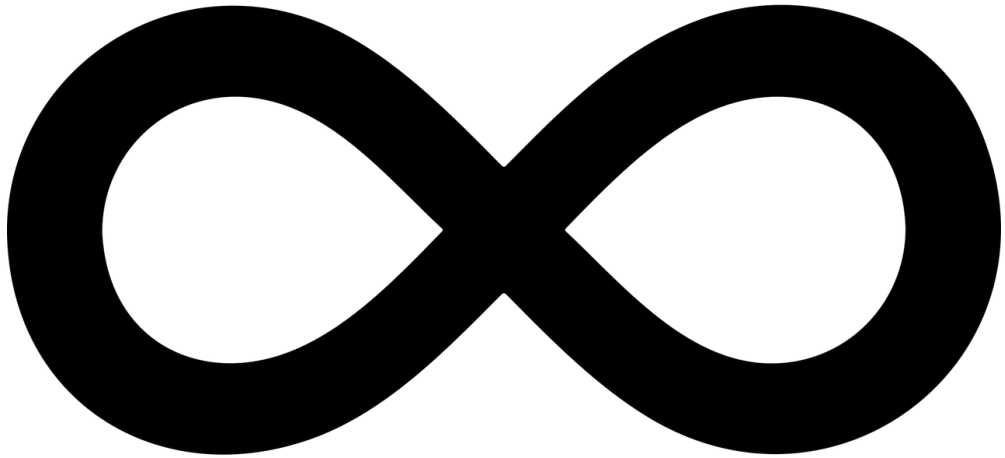
$$f'(x) = f(x)$$

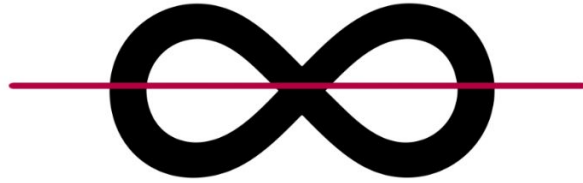


Simetrias no Plano



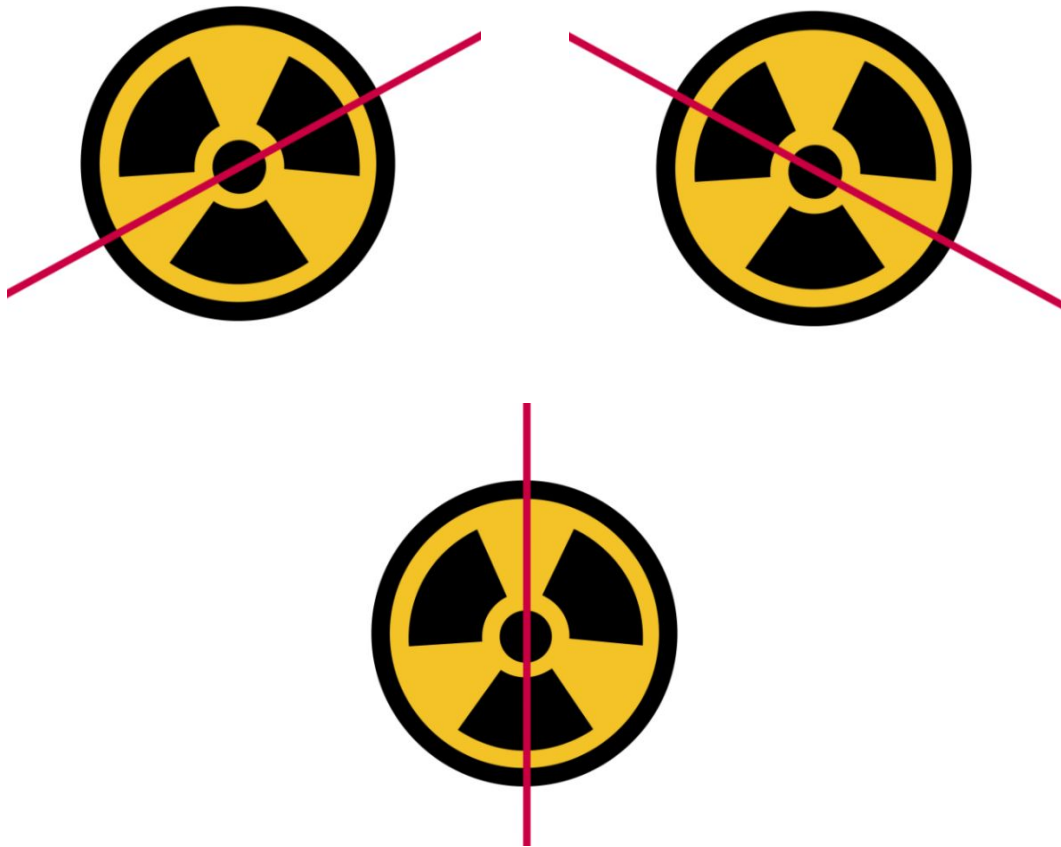




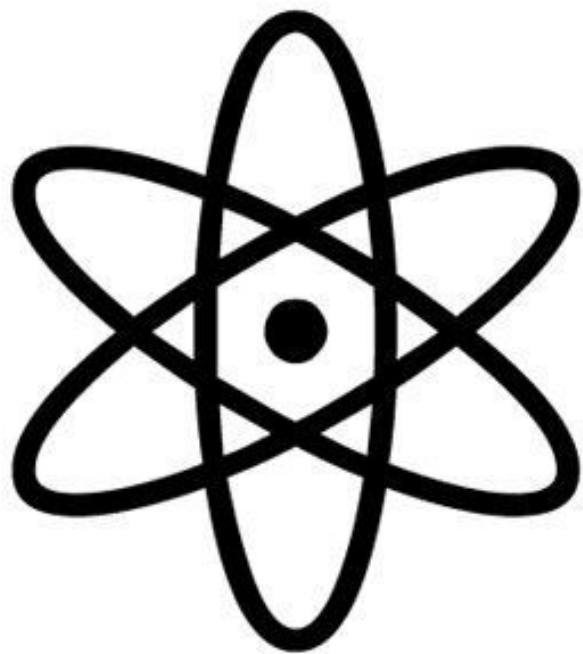


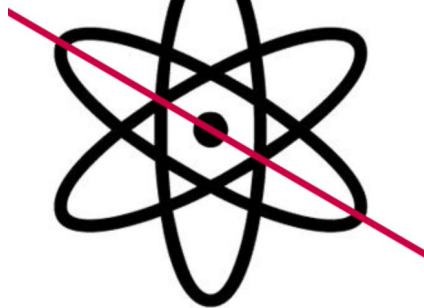
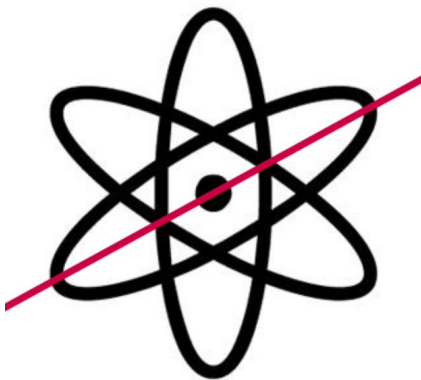
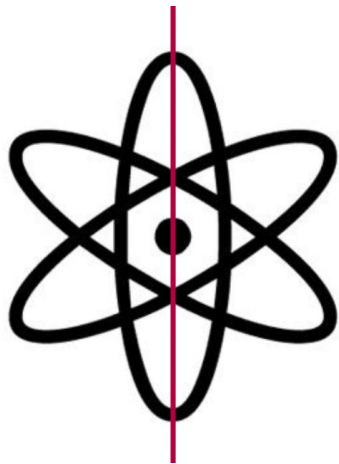
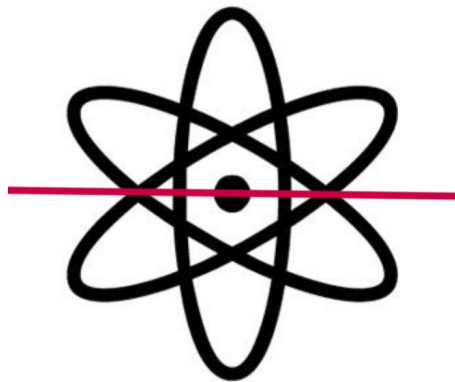
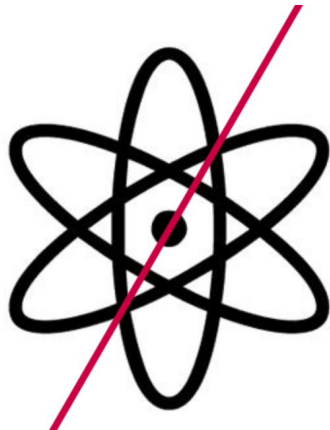
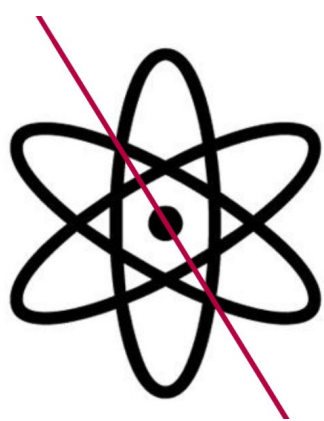
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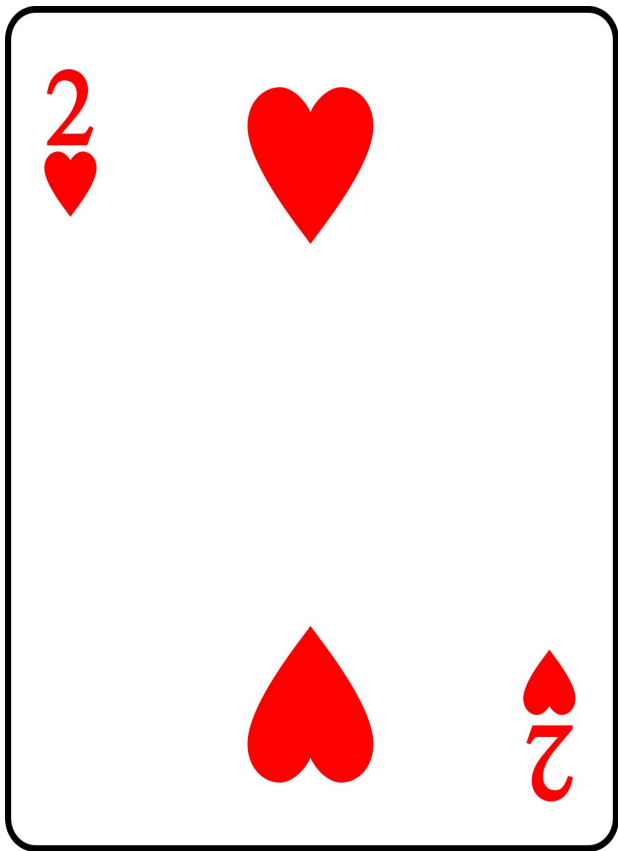


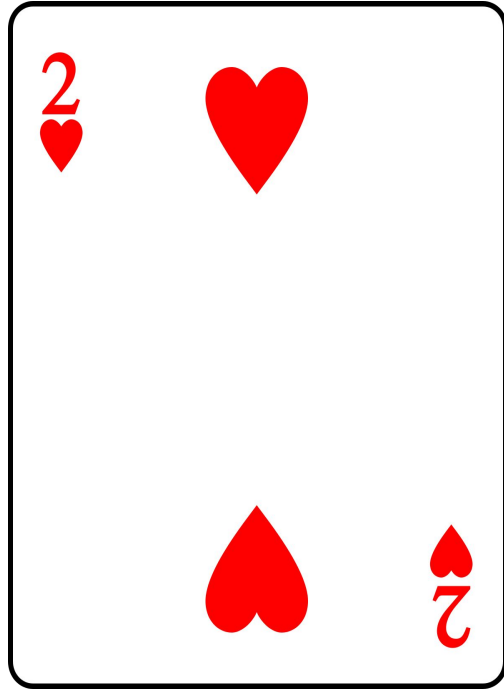
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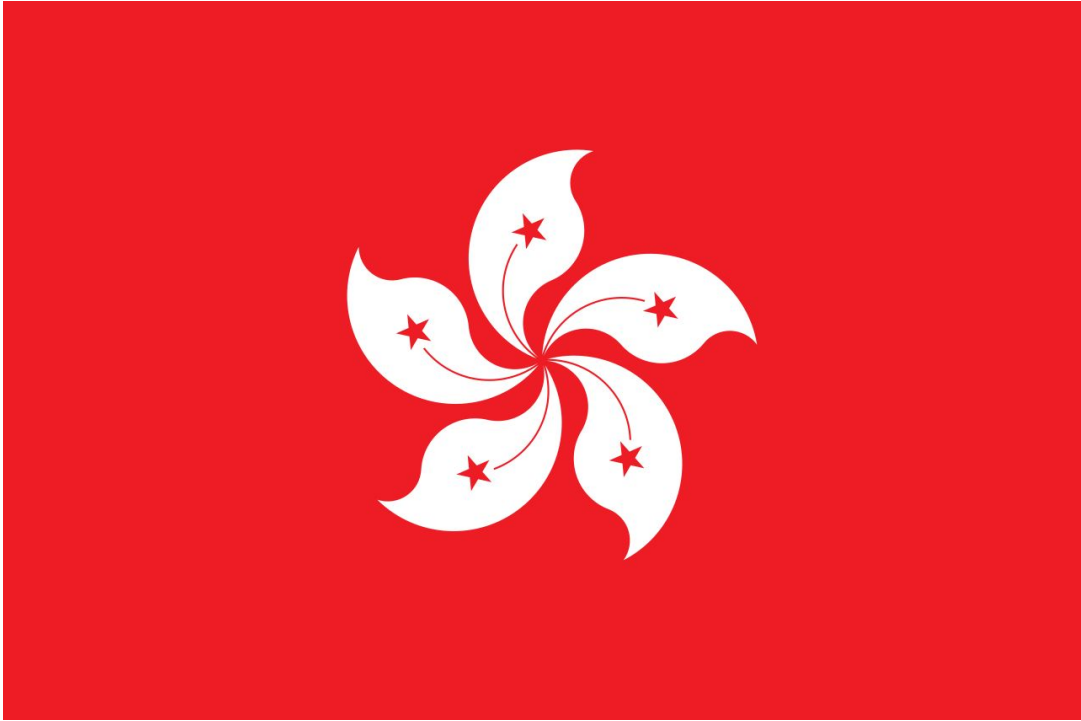


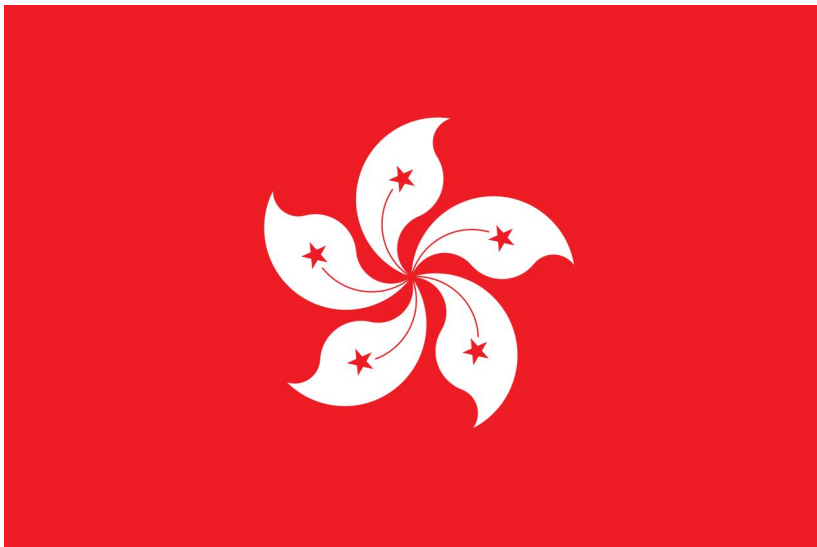
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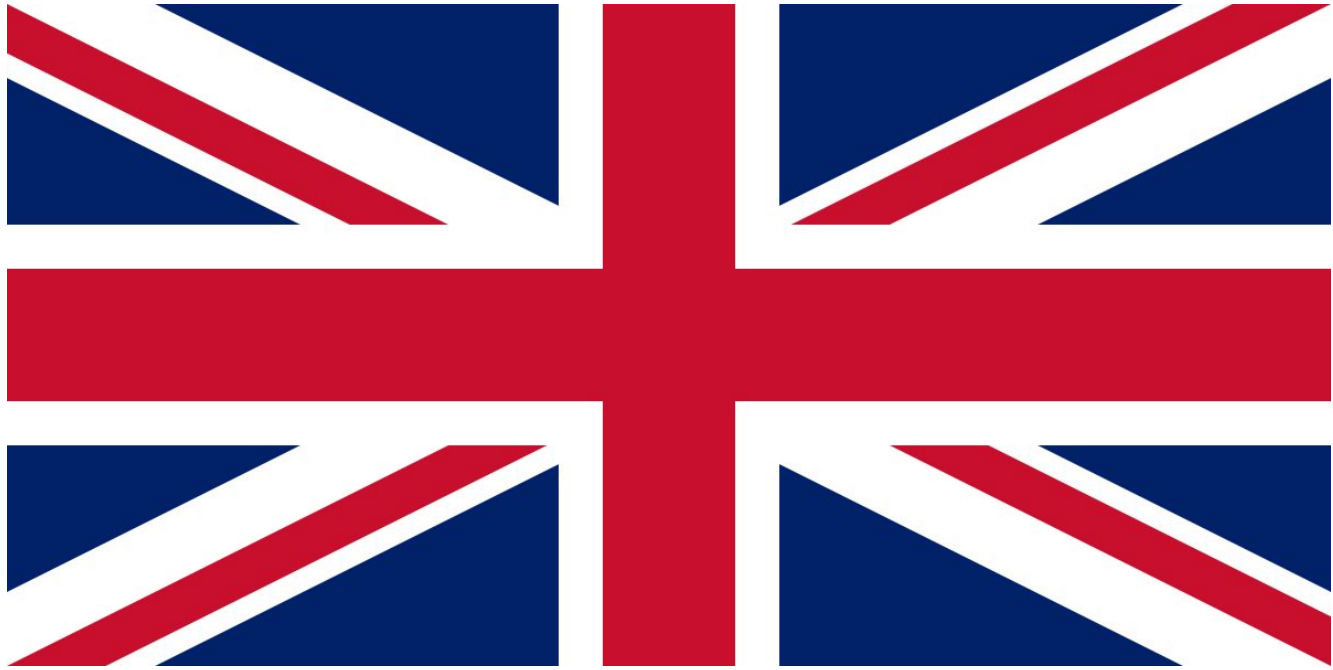


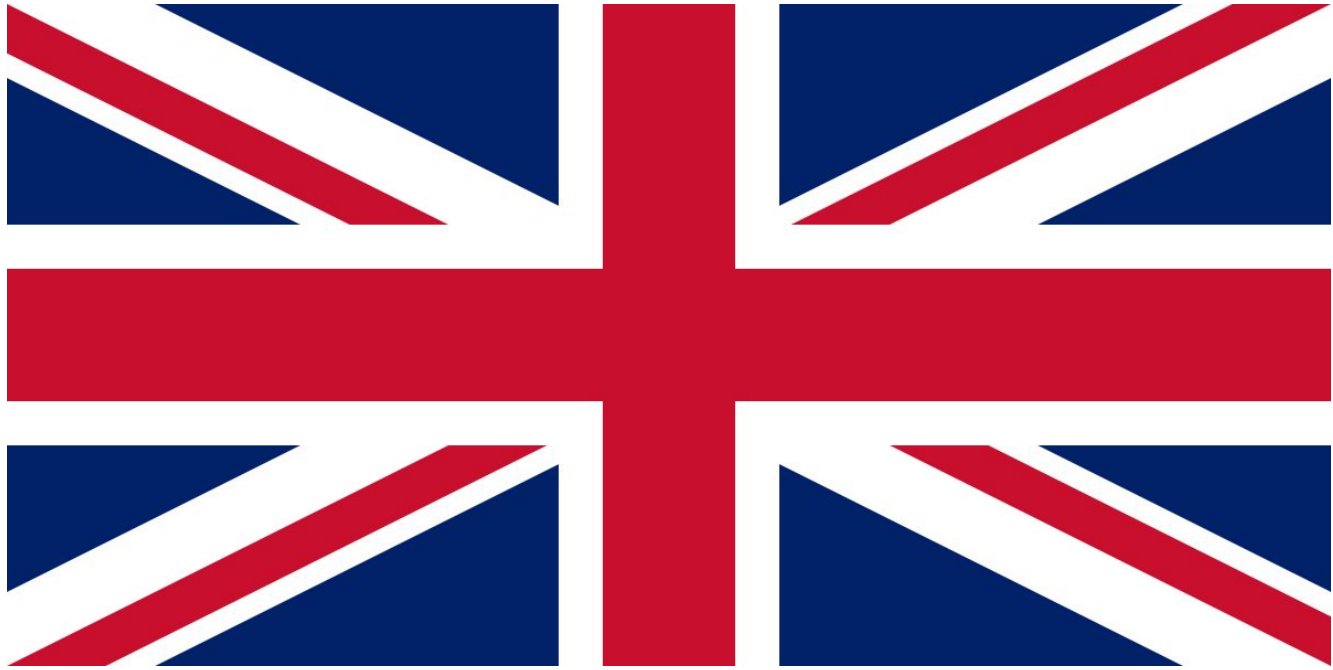
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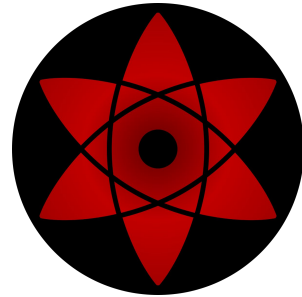
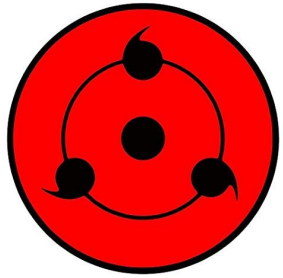
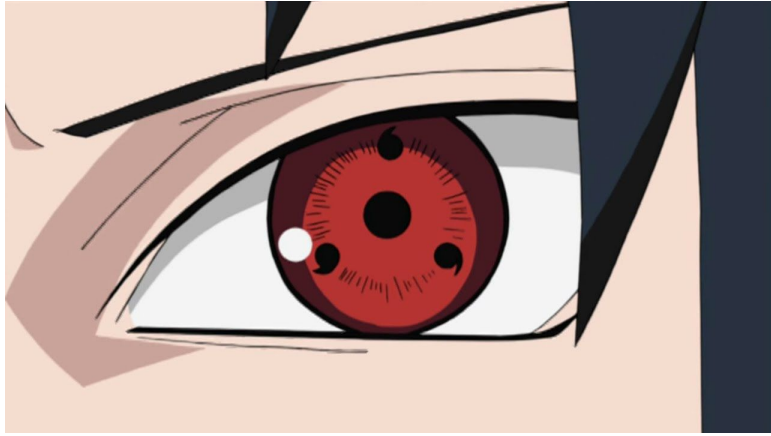


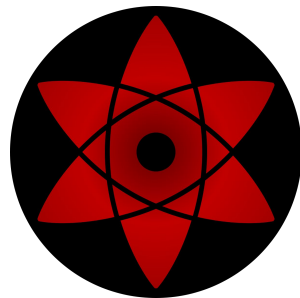
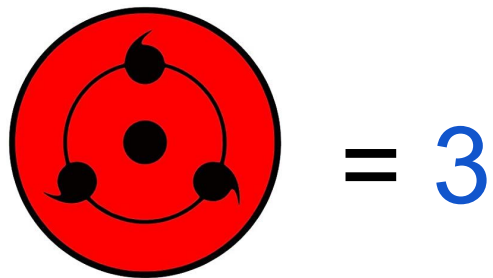
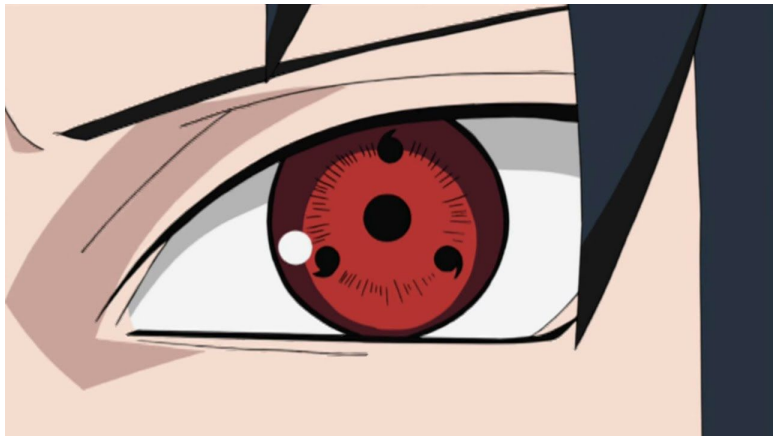
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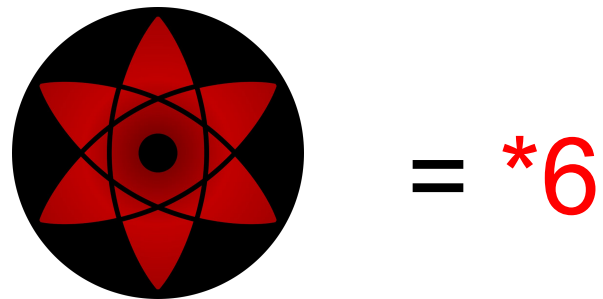
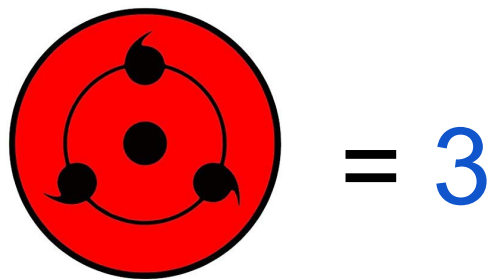
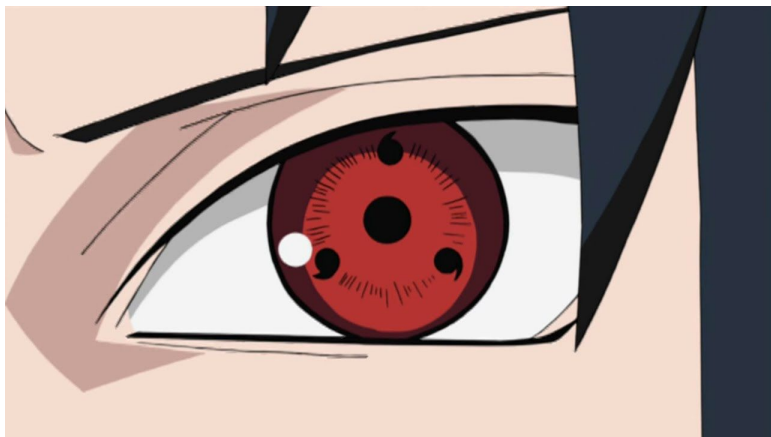


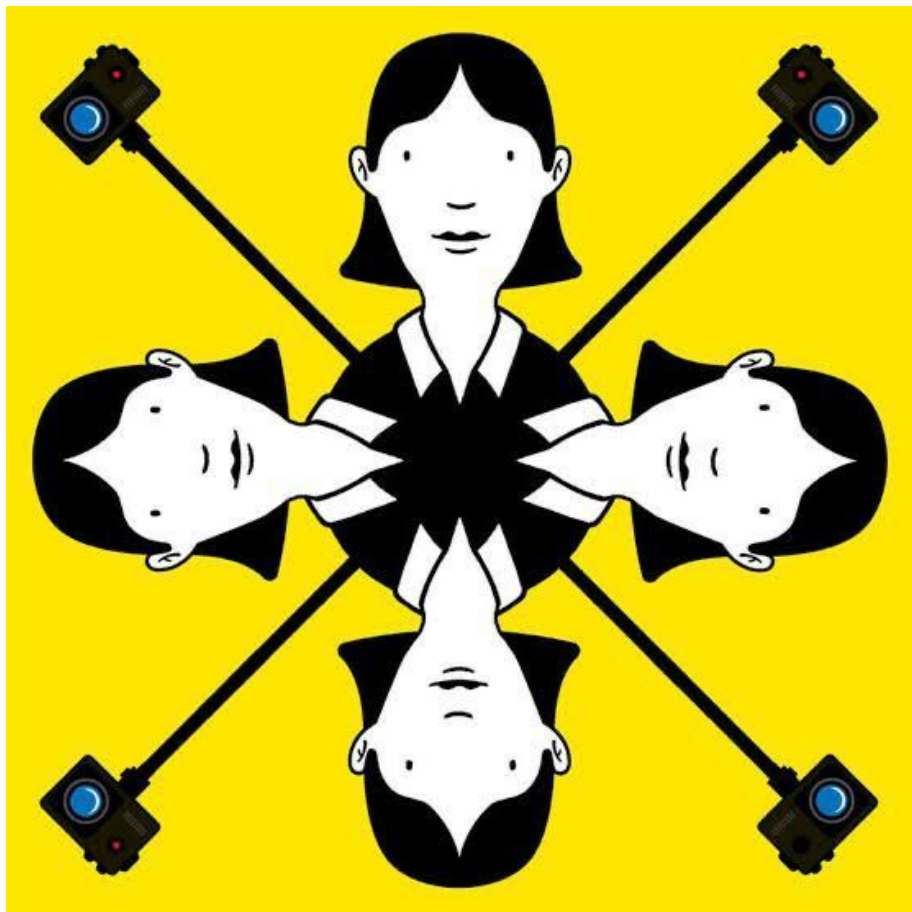


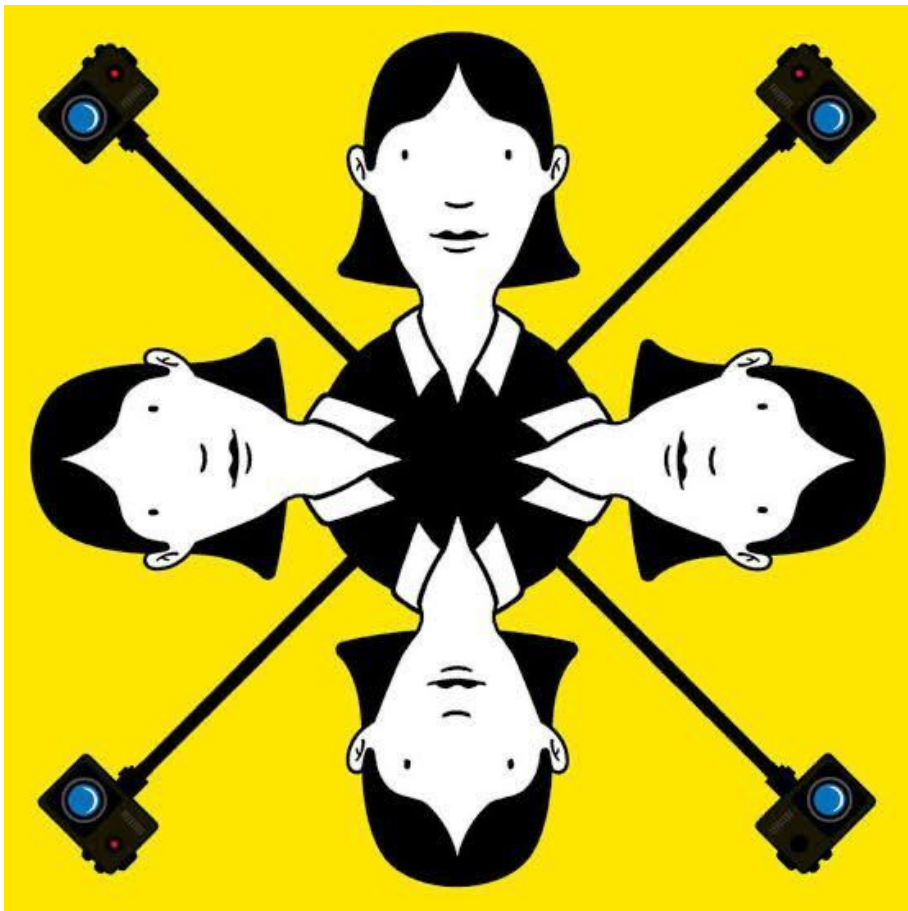
2





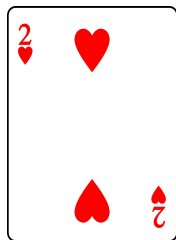




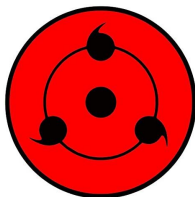


Não tem!

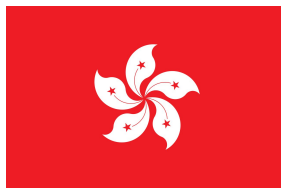
Até agora...



2



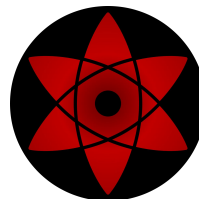
3



5



*



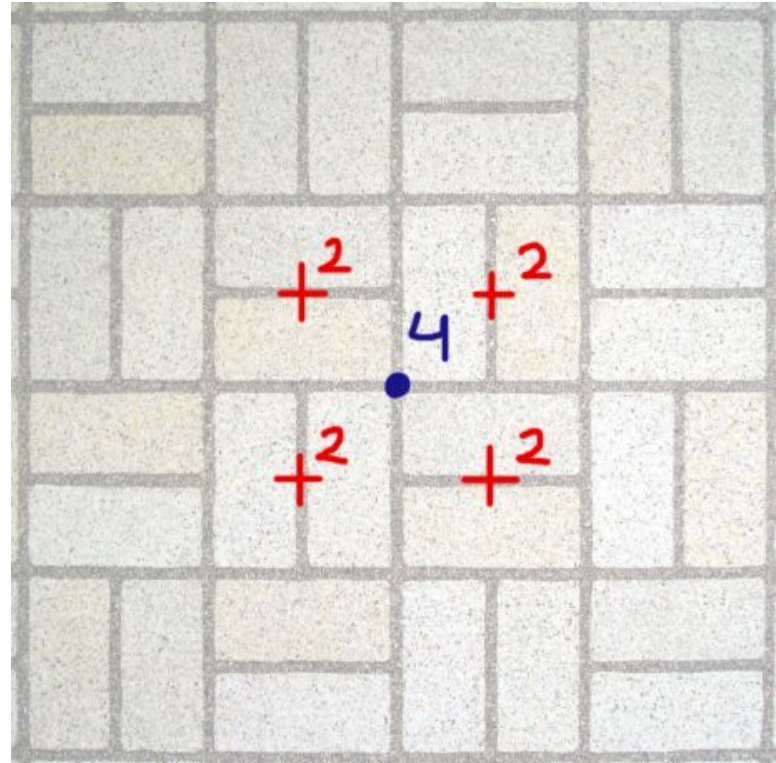
*6



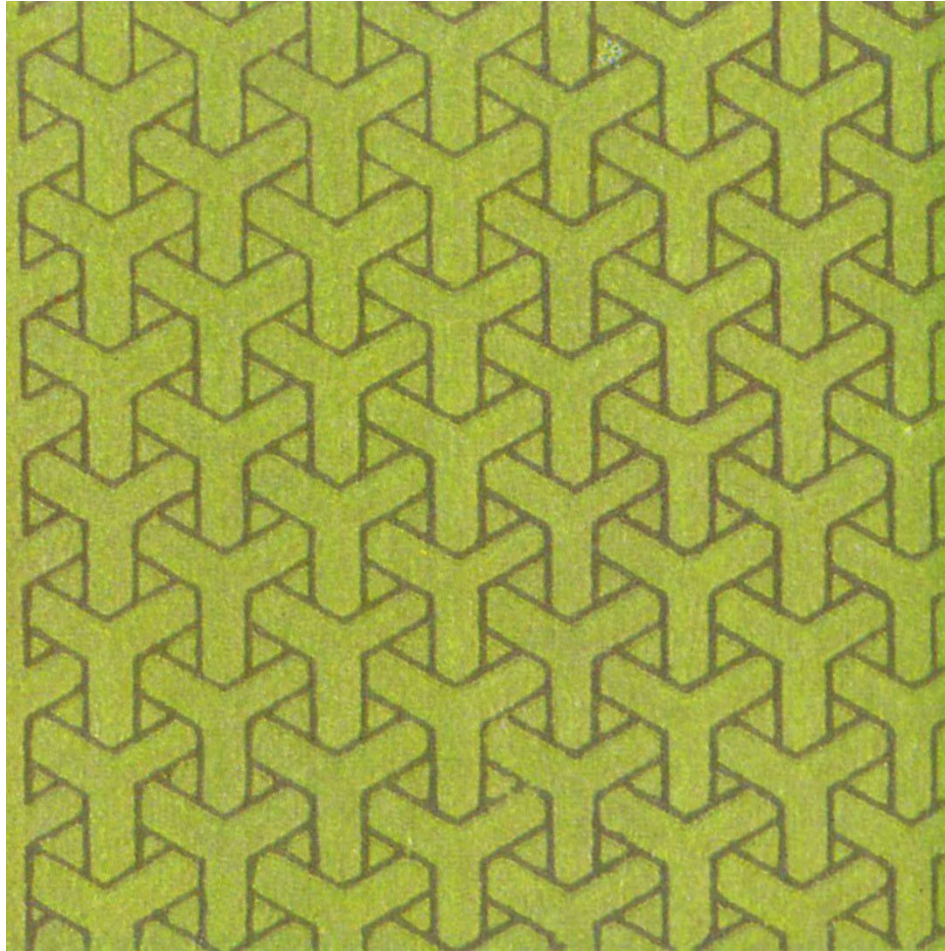
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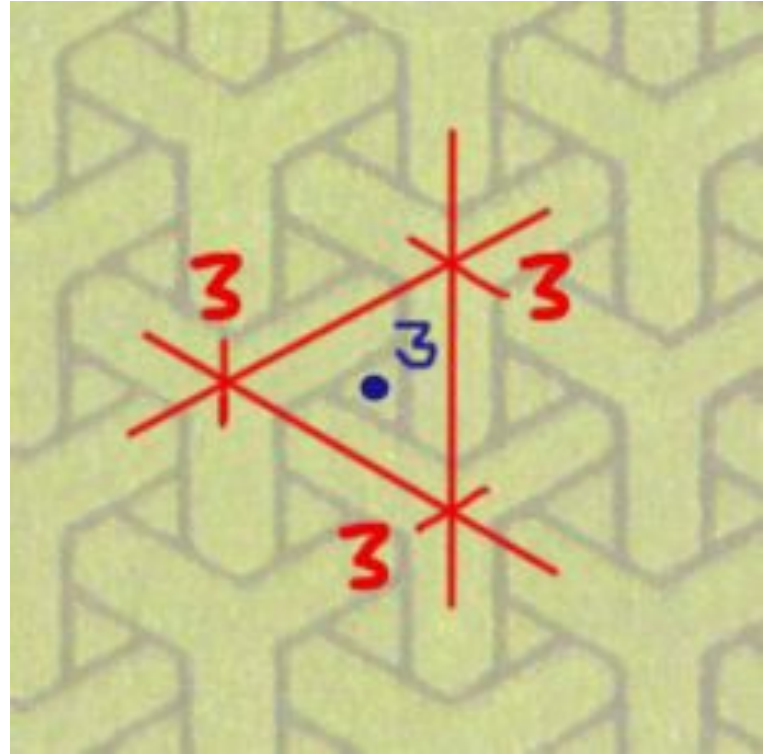
Padrões infinitos



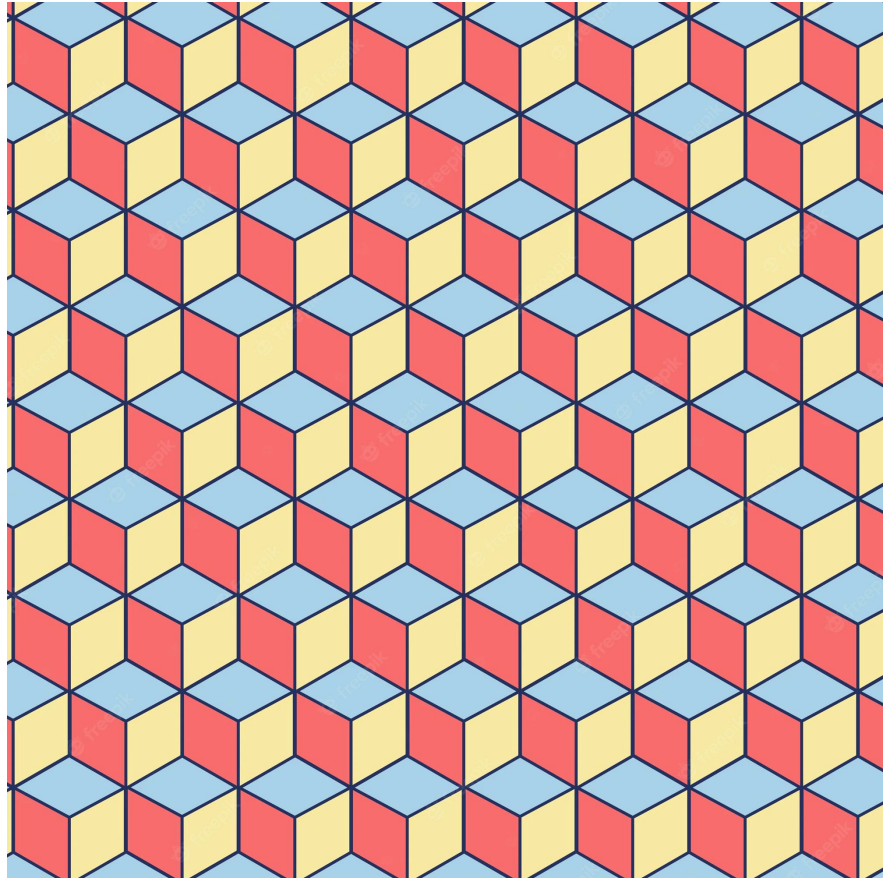


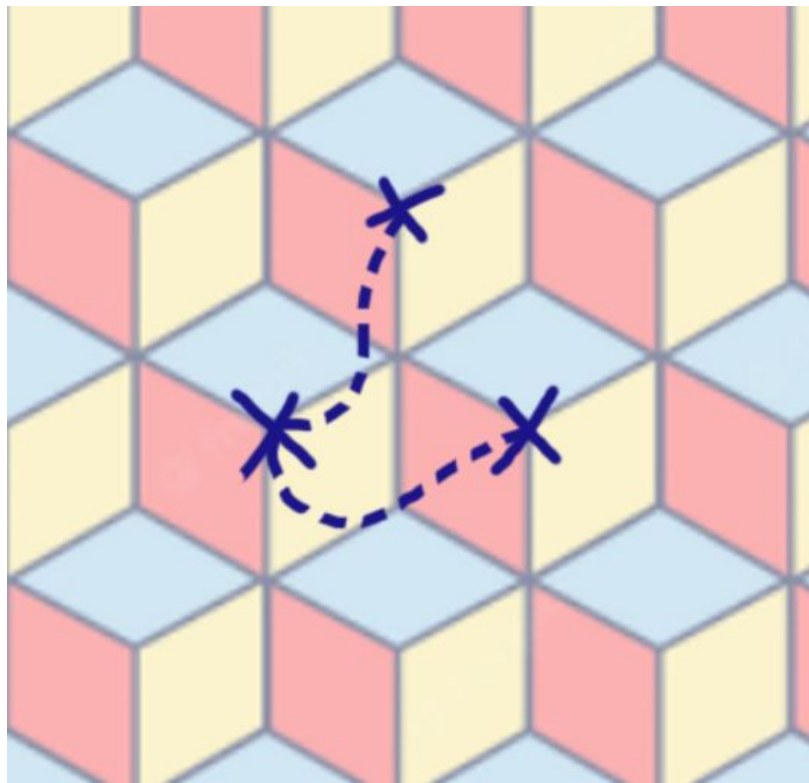
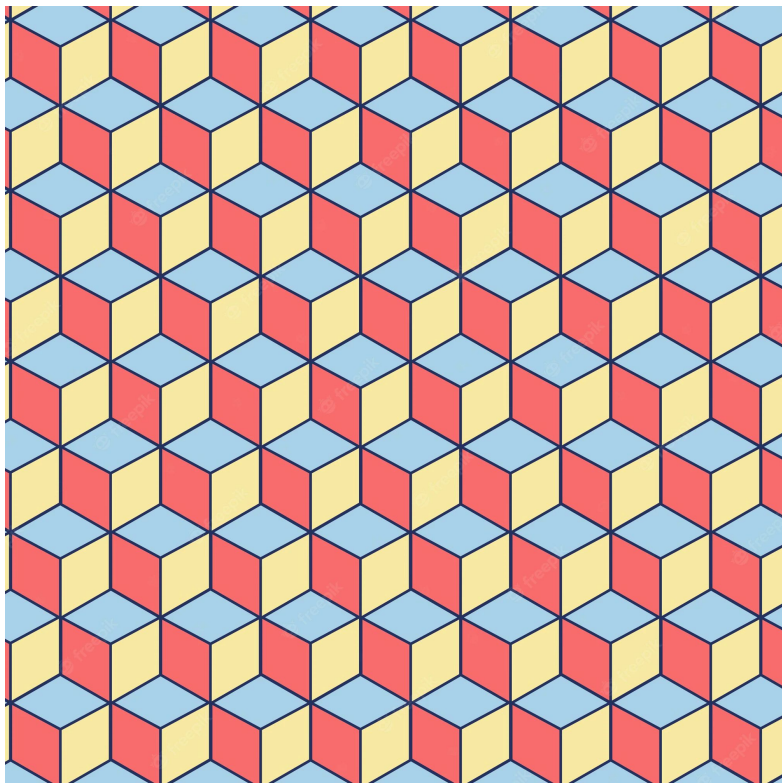
$$4 * 2$$





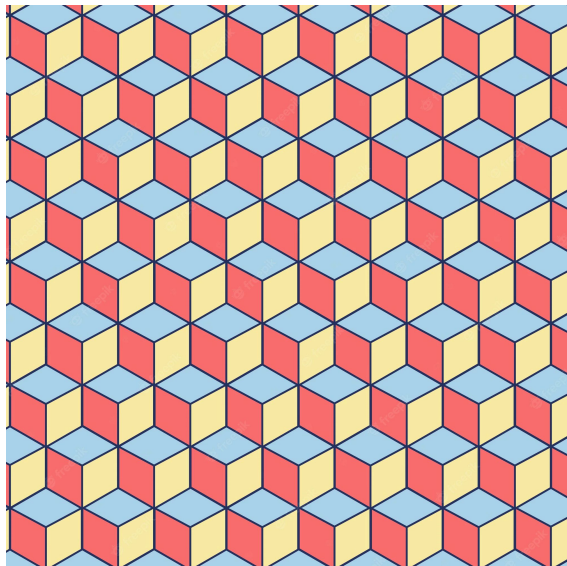
$$3^*3$$



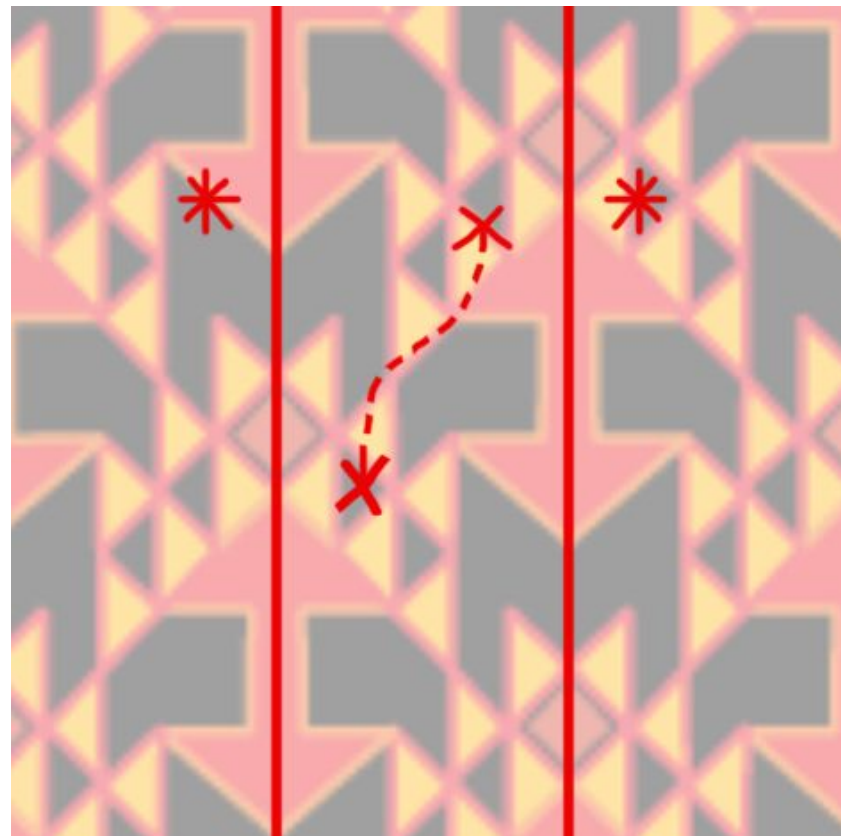


O

Wonders







*X

Miracles



XX



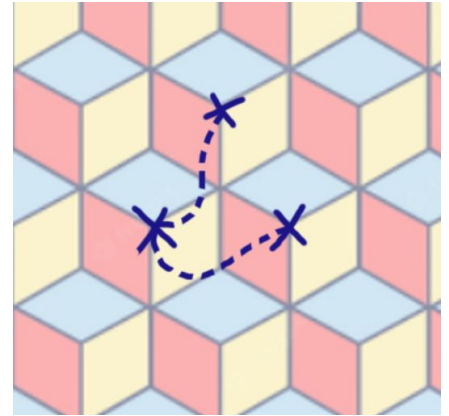
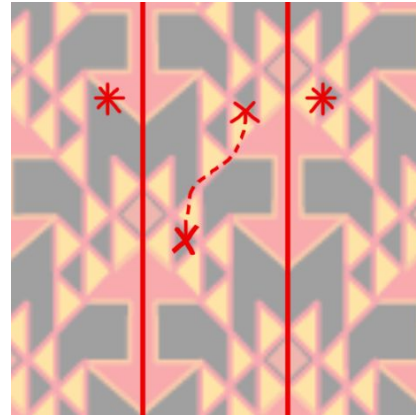
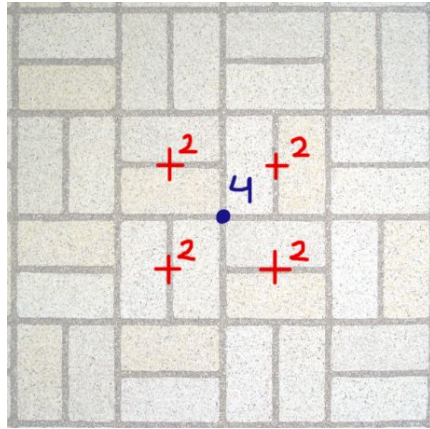
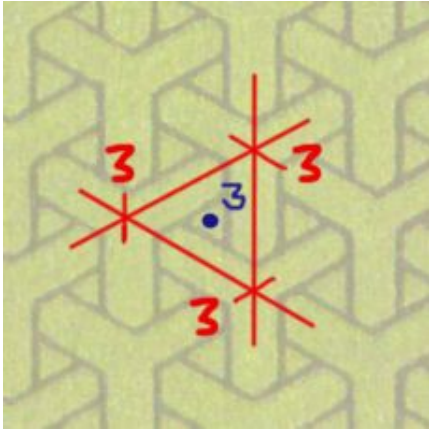
22X



*X

Até agora...

Reflexões: *N Miracles: X
Rotações: N Wonders: O



Magic Theorem

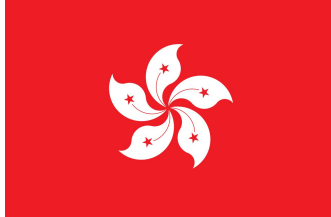
Magic Theorem

Símbolo	Custo
○	2
*	1
×	1
n	$\frac{n-1}{n}$
n	$\frac{n-1}{2n}$

Magic Theorem



$$\text{Custo}(*3) = 1 + \frac{2}{6} = \frac{4}{3}$$



$$\text{Custo}(5) = \frac{4}{5}$$



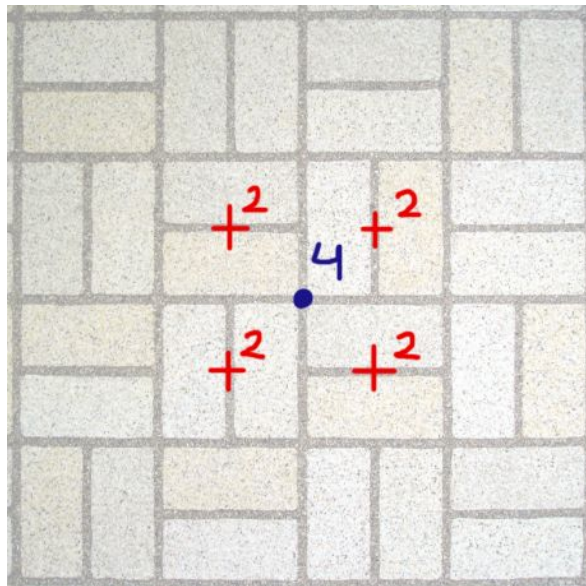
$$\text{Custo}(*) = 1$$

Símbolo	Custo
○	2
*	1
×	1
n	$\frac{n-1}{n}$
n	$\frac{n-1}{2n}$

Enunciado...

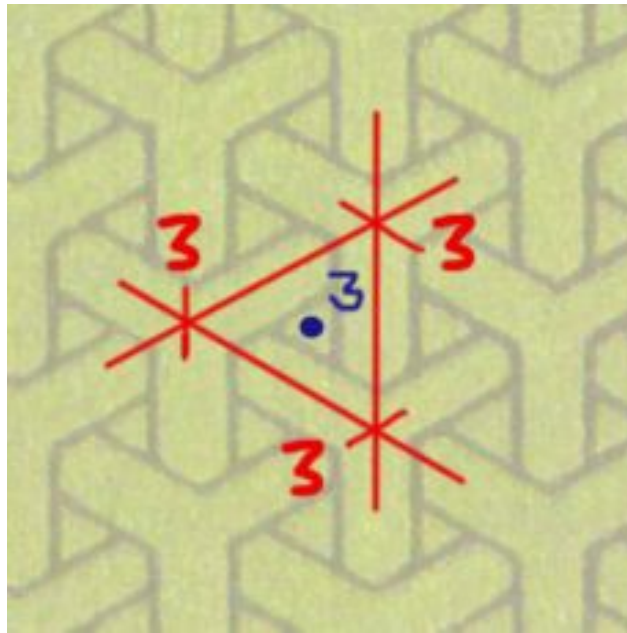
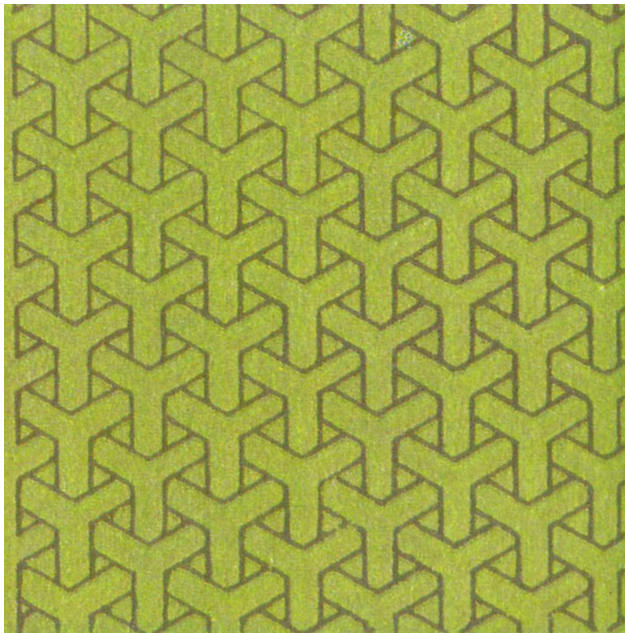
Magic Theorem

O custo de toda simetria no plano infinito tem soma 2.



Símbolo	Custo
○	2
*	1
×	1
n	$\frac{n-1}{n}$
n	$\frac{n-1}{2n}$

$$\text{Custo}(4^*2) = \frac{3}{4} + 1 + \frac{1}{4} = 2$$



Símbolo	Custo
○	2
*	1
×	1
n	$\frac{n-1}{n}$
n	$\frac{n-1}{2n}$

$$\text{Custo}(3^*3) = \frac{2}{3} + 1 + \frac{1}{3} = 2$$



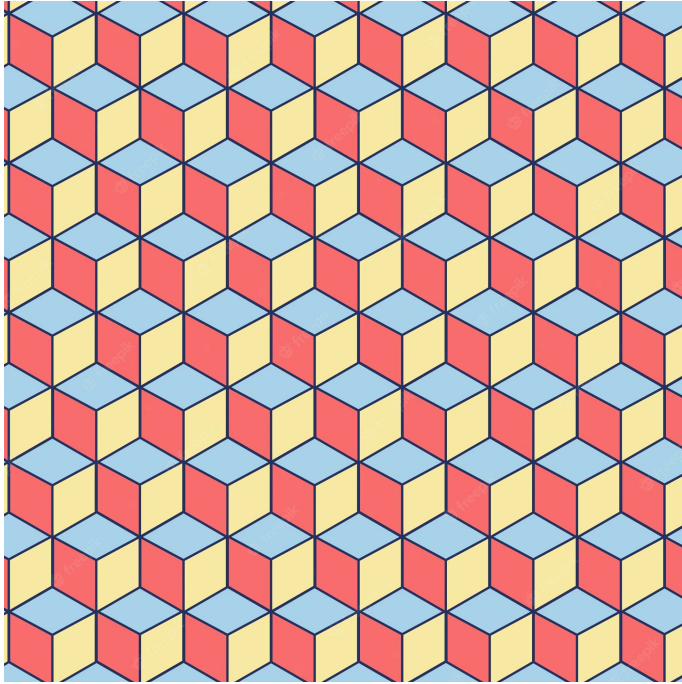
Símbolo	Custo
○	2
*	1
×	1
n	$\frac{n-1}{n}$
n	$\frac{n-1}{2n}$

22X

$$\frac{1}{2} + \frac{1}{2} + 1 = 2$$

*X

$$1 + 1 = 2$$



Símbolo	Custo
○	2
*	1
×	1
n	$\frac{n-1}{n}$
n	$\frac{n-1}{2n}$

$$\text{Custo}(\text{O}) = 2$$

Todas as simetrias...

Símbolo	Custo
○	2
*	1
×	1
n	$\frac{n-1}{n}$
n	$\frac{n-1}{2n}$

Todas as simetrias...

○

XX

*X

**

Símbolo	Custo
○	2
*	1
×	1
n	$\frac{n-1}{n}$
n	$\frac{n-1}{2n}$

Todas as simetrias...

O

XX

*X

**

632, 442,
333, 2222

Símbolo	Custo
o	2
*	1
×	1
n	$\frac{n-1}{n}$
n	$\frac{n-1}{2n}$

Todas as simetrias...

O

632, 442,
333, 2222

XX

*X

**

*632, *442,
*333, *2222

Símbolo	Custo
o	2
*	1
×	1
n	$\frac{n-1}{n}$
n	$\frac{n-1}{2n}$

Todas as simetrias...

O

632, 442,
333, 2222

2*22 4*2 3*3 22* 22X

XX *X **

*632, *442,
*333, *2222

Símbolo	Custo
o	2
*	1
x	1
n	$\frac{n-1}{n}$
n	$\frac{n-1}{2n}$

Wallpaper Groups

O

XX

*X

**

632, 442,
333, 2222

*632, *442,
*333, *2222

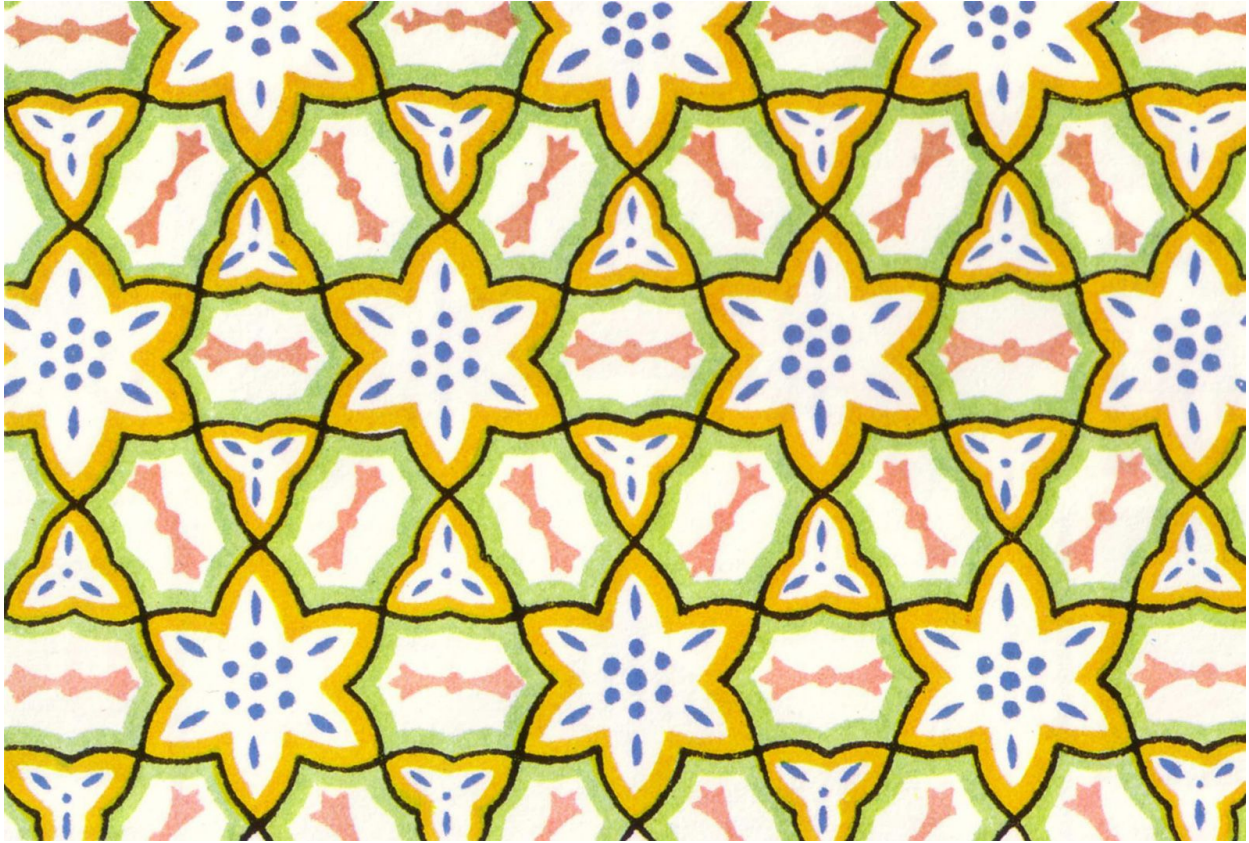
2*22

4*2

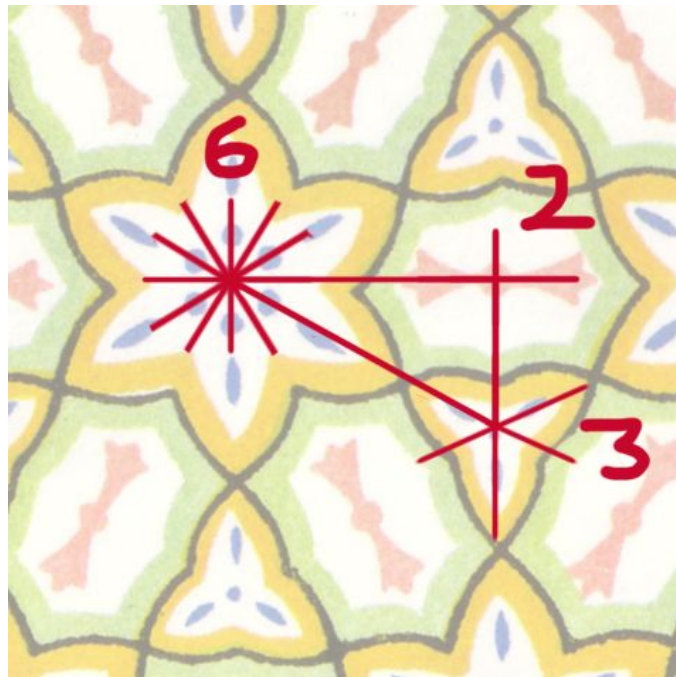
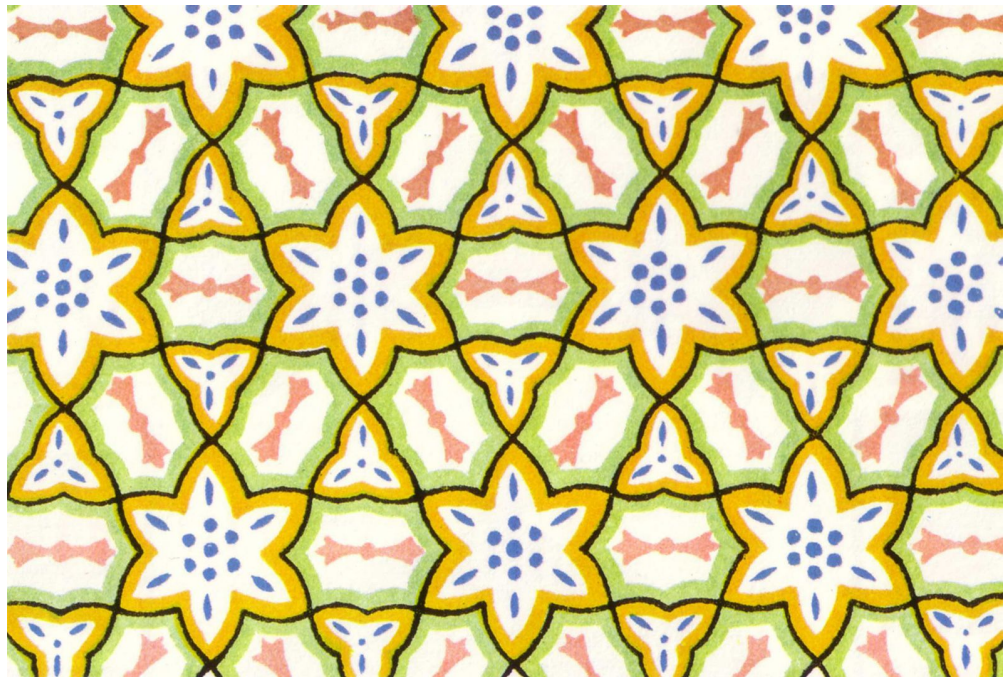
3*3

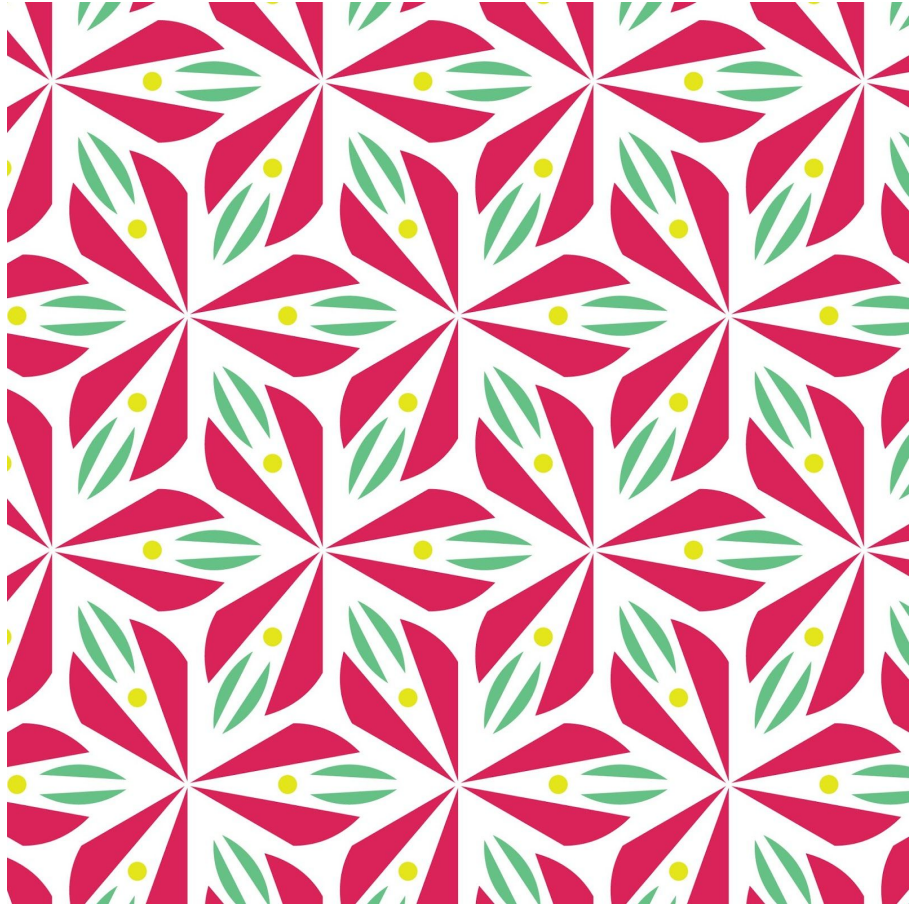
22*

22X

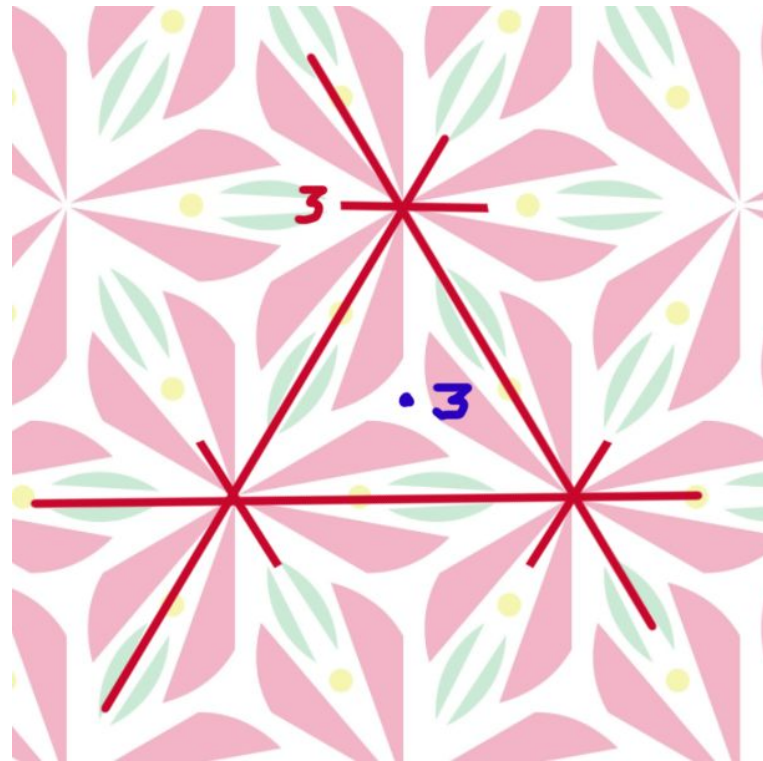
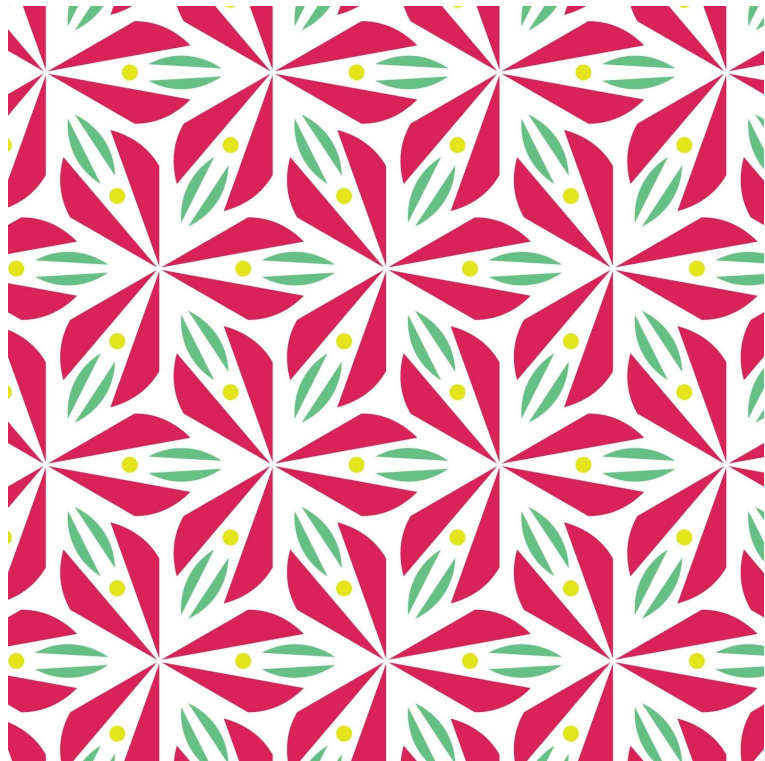


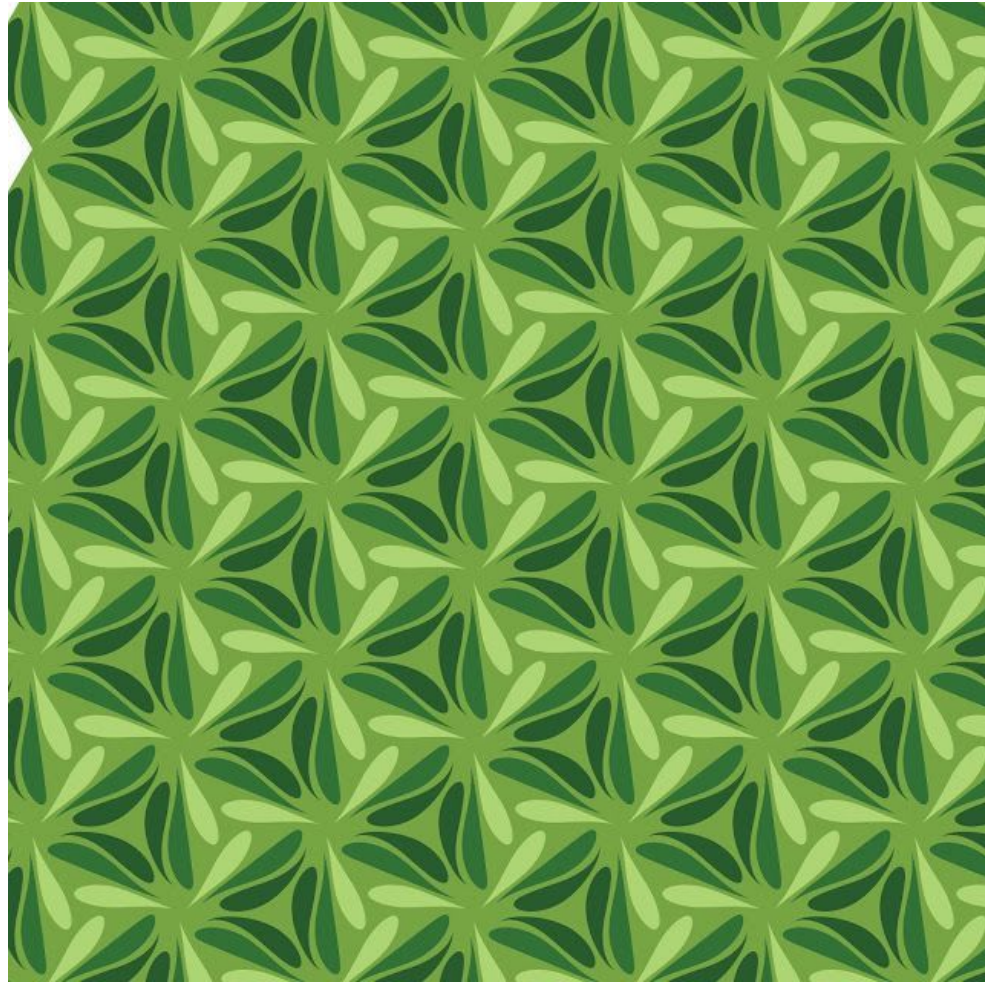
*632



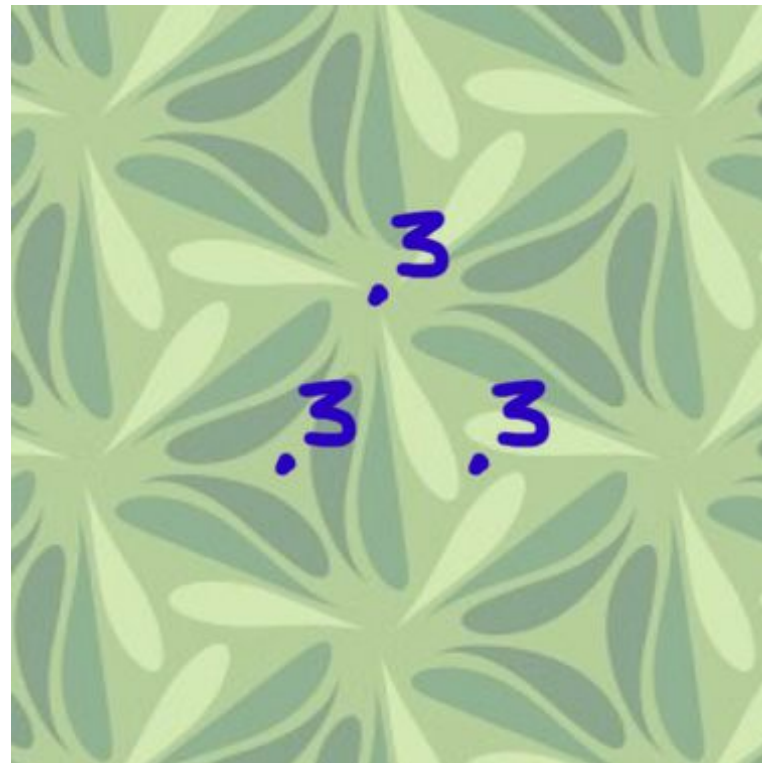
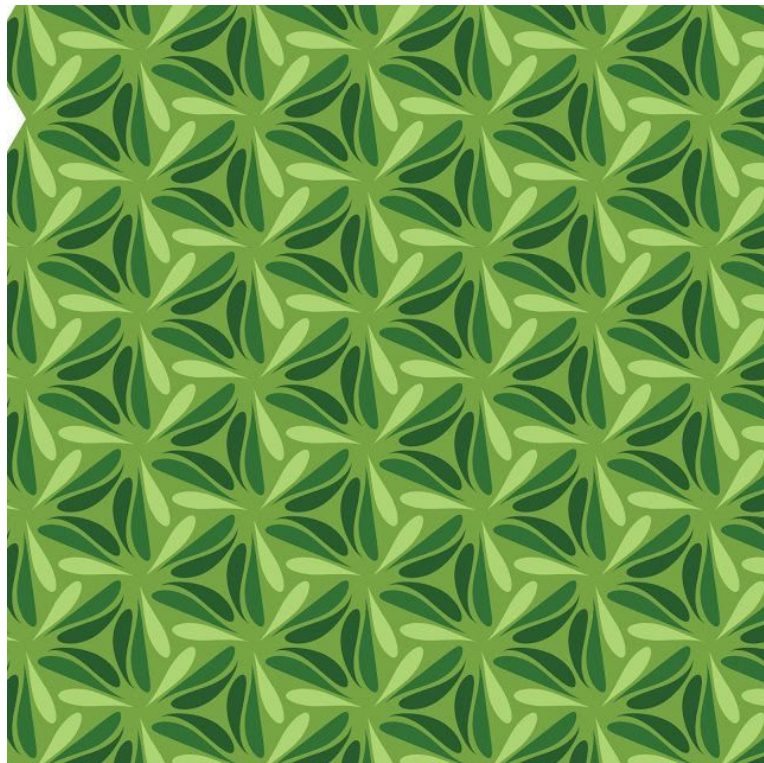


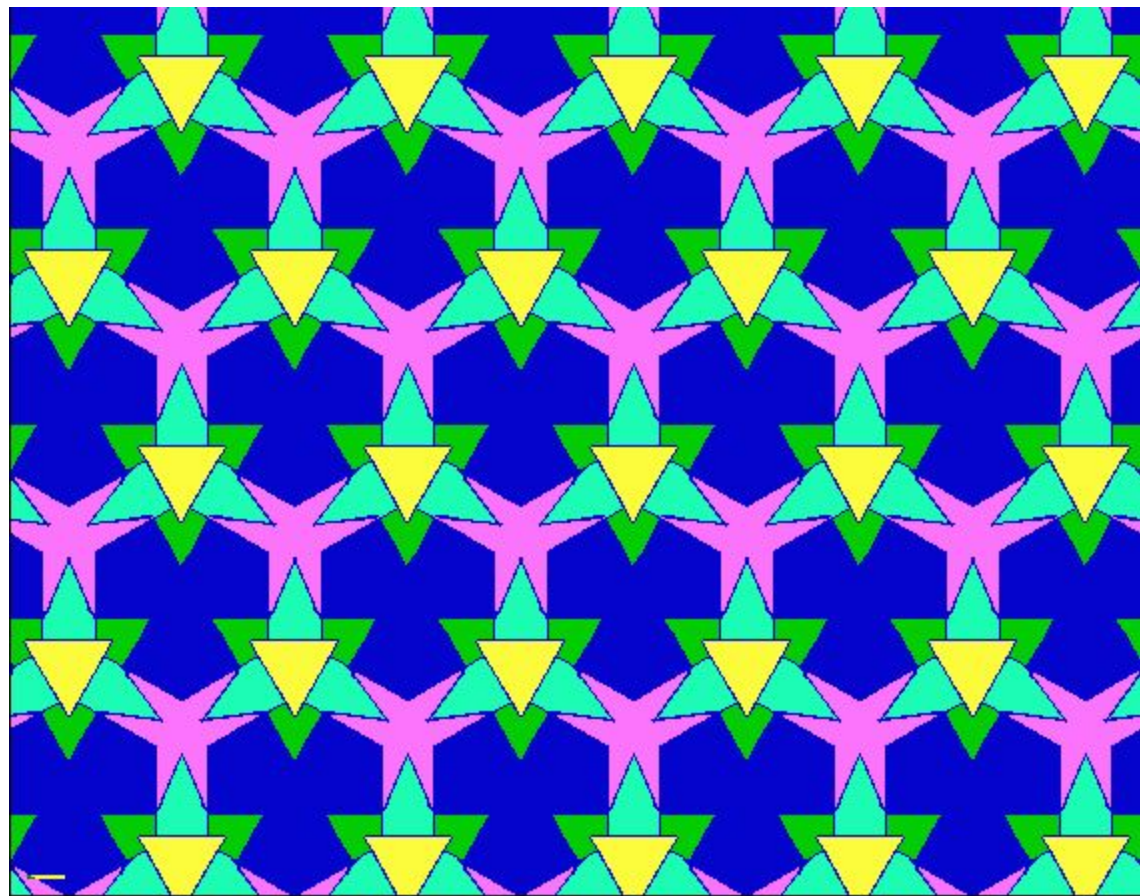
3*3



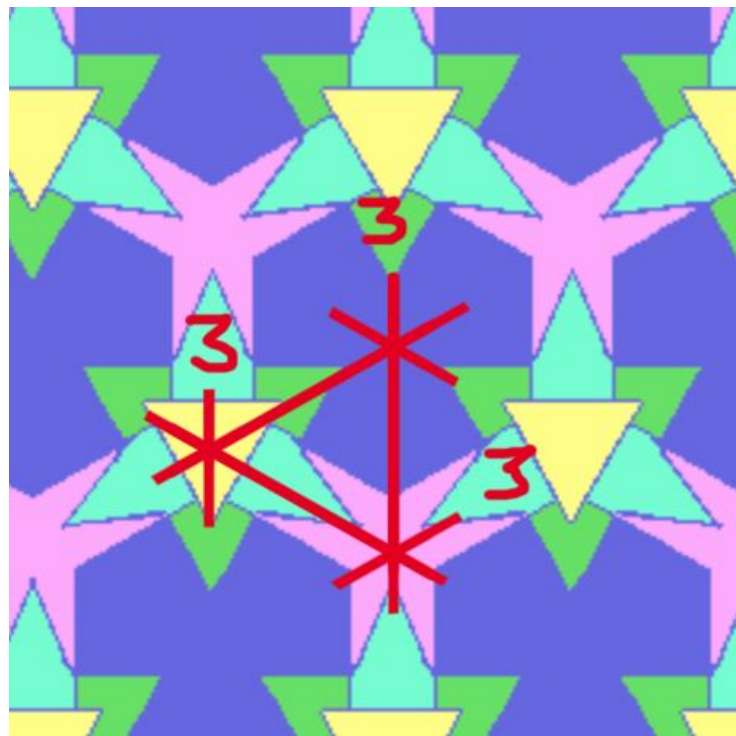
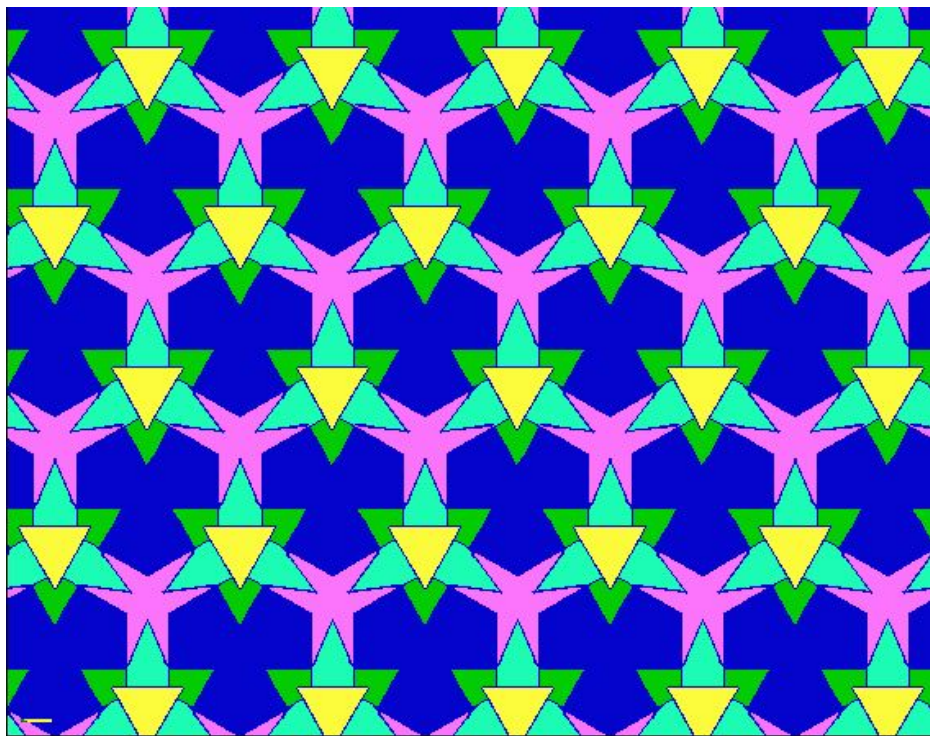


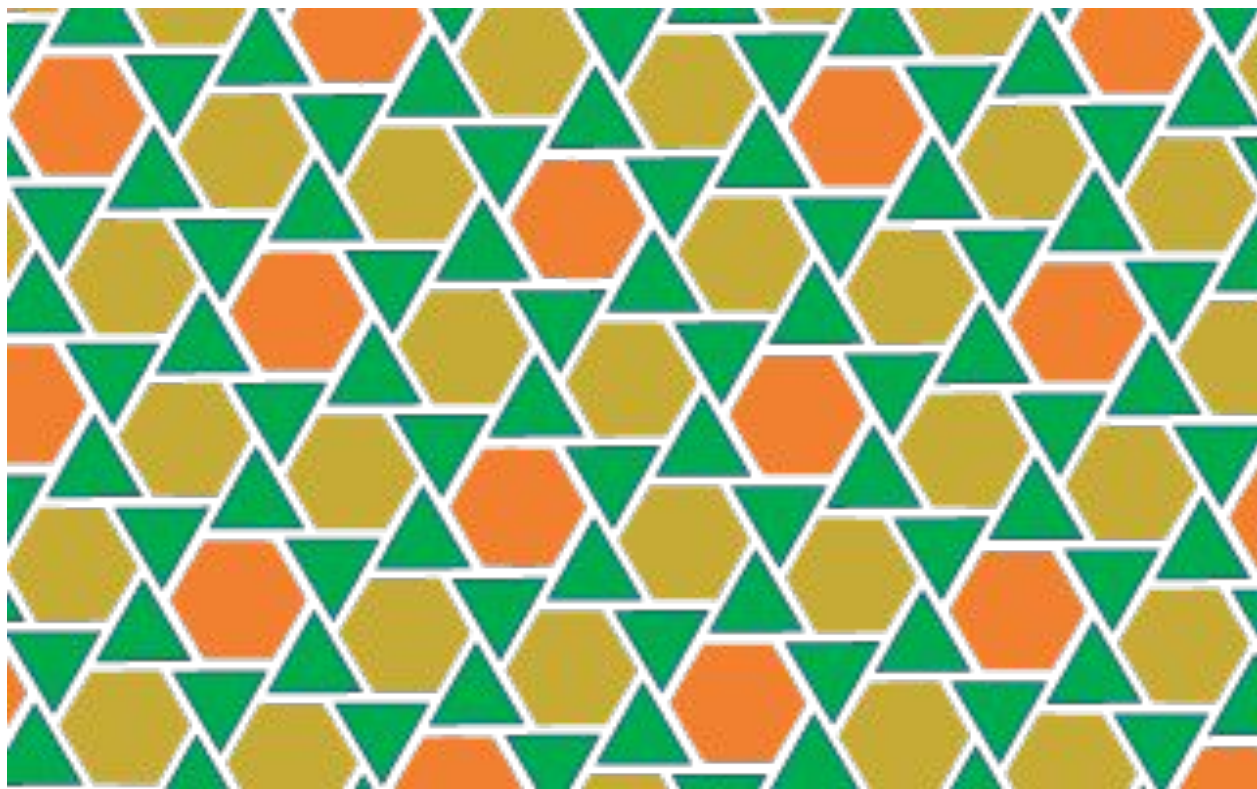
333



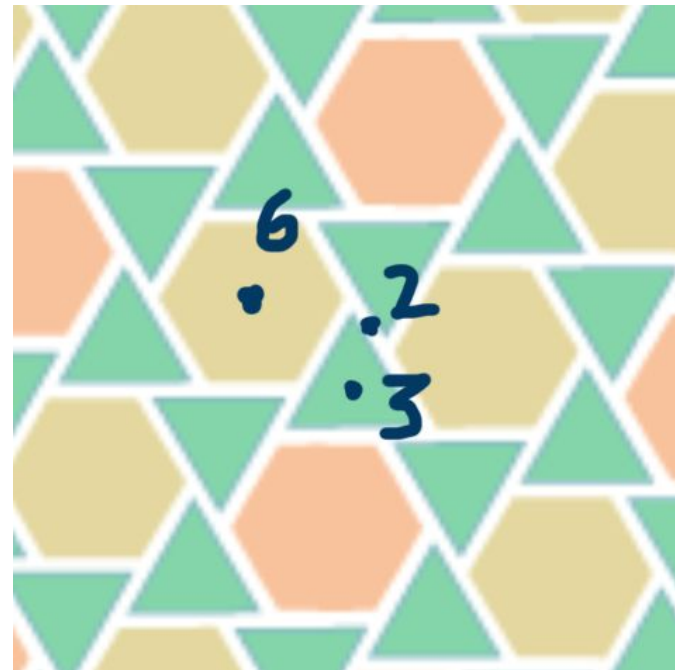
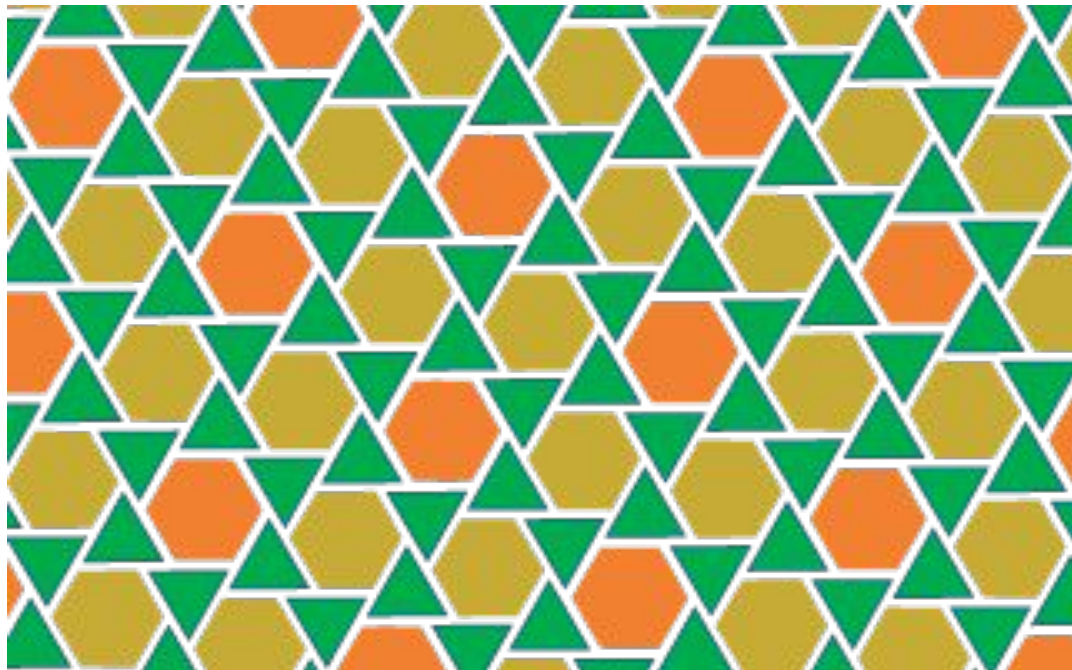


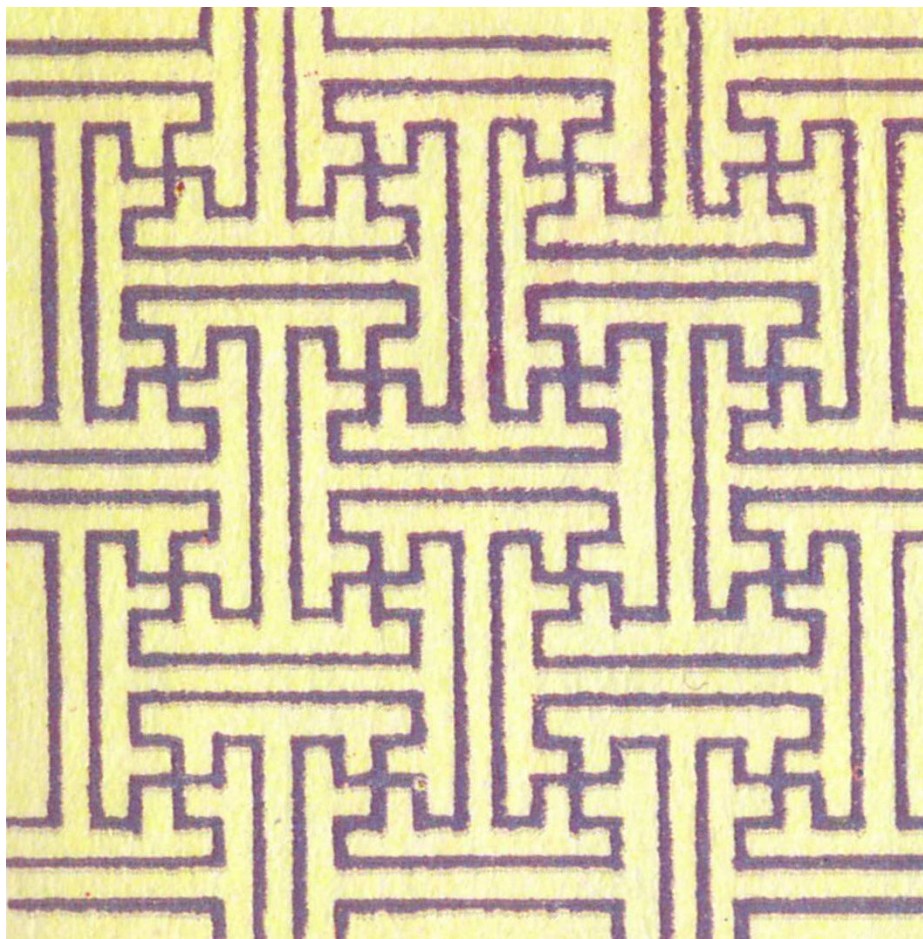
*333



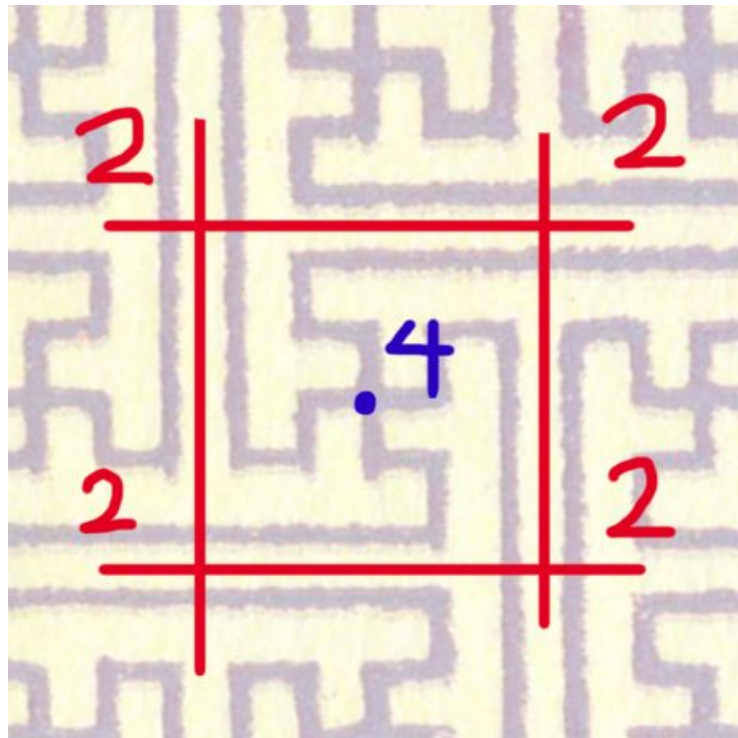
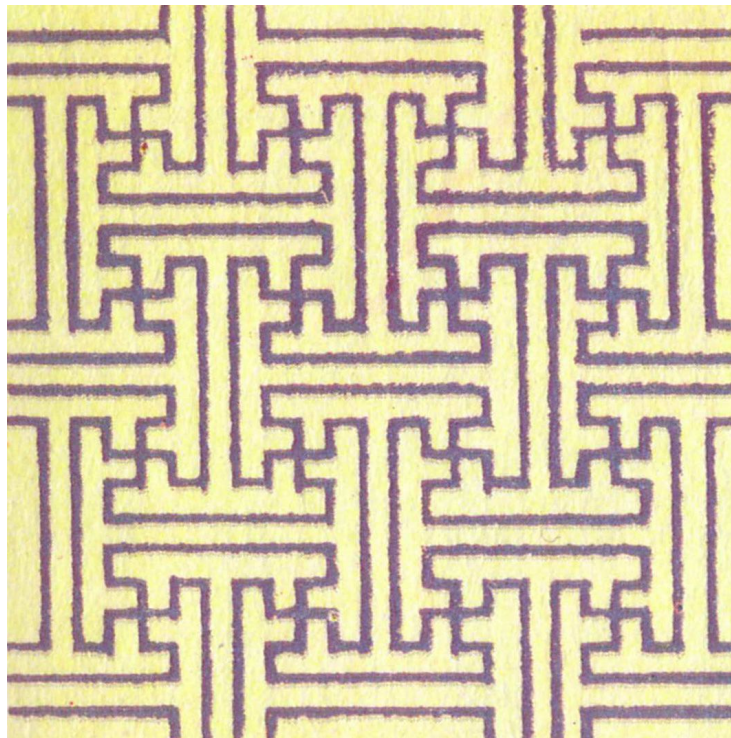


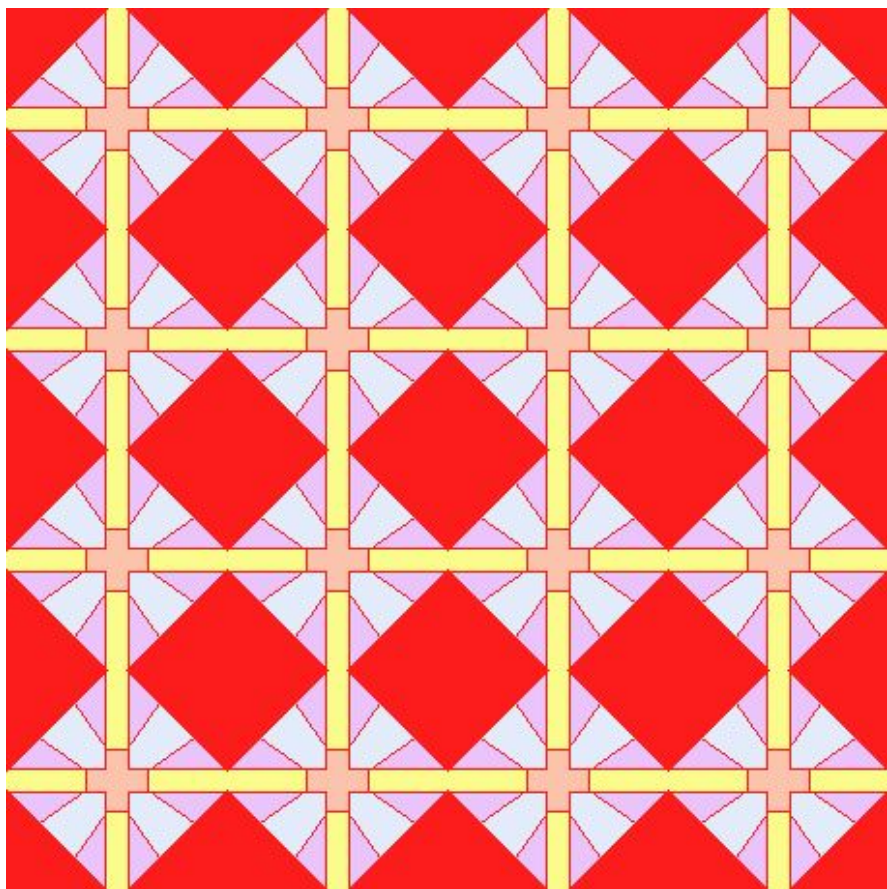
632



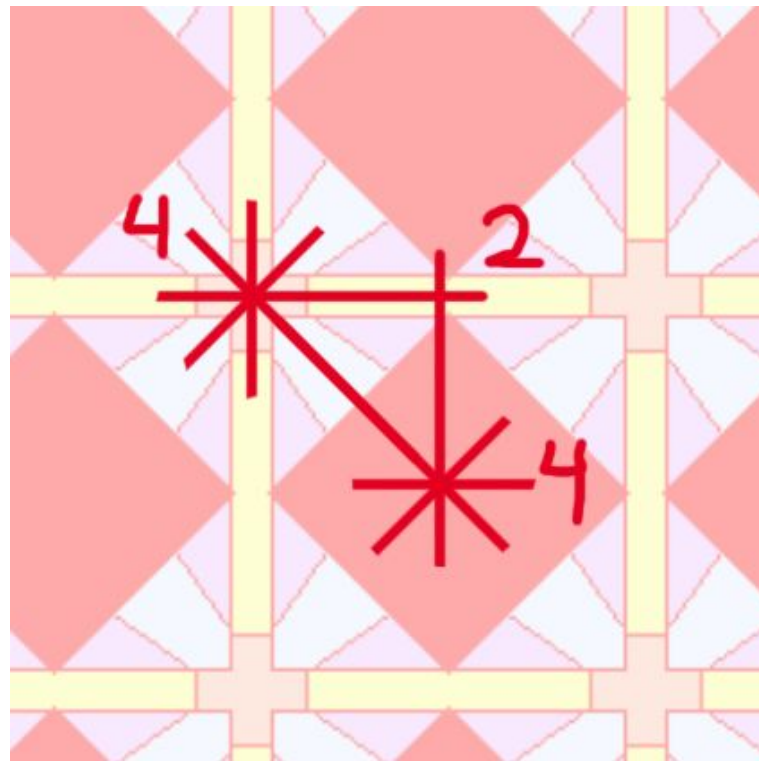
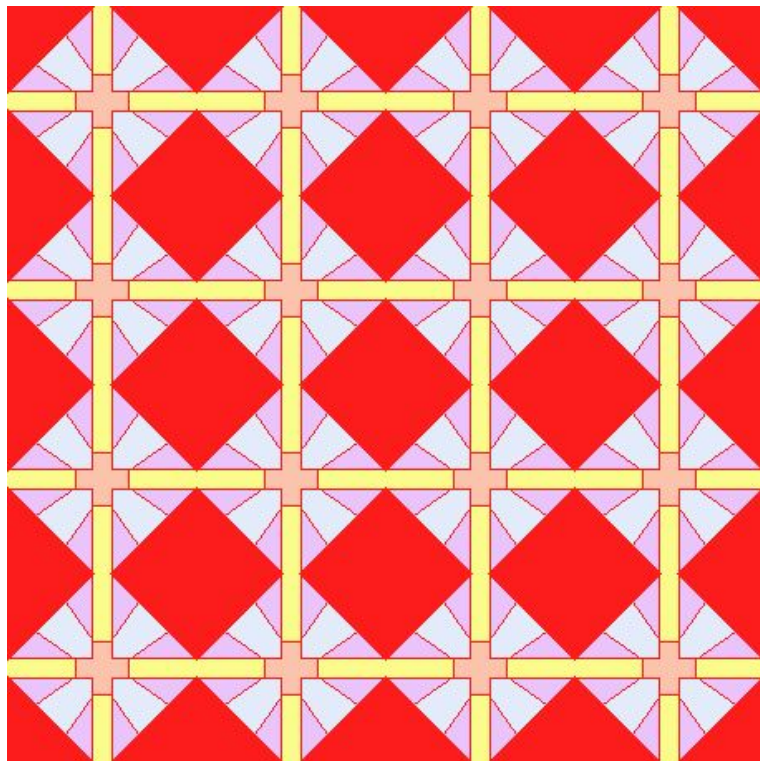


4×2



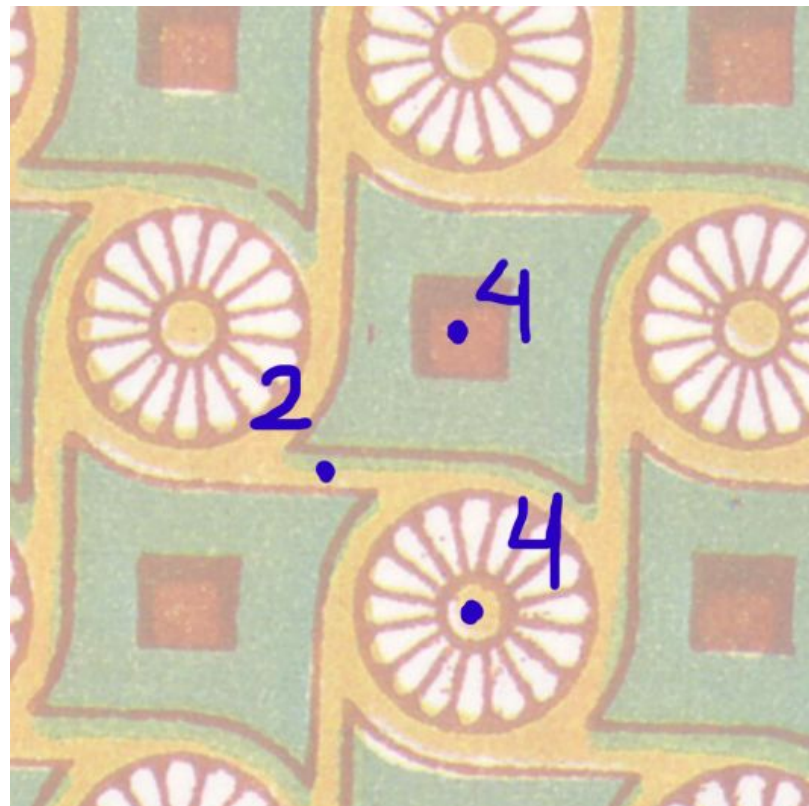


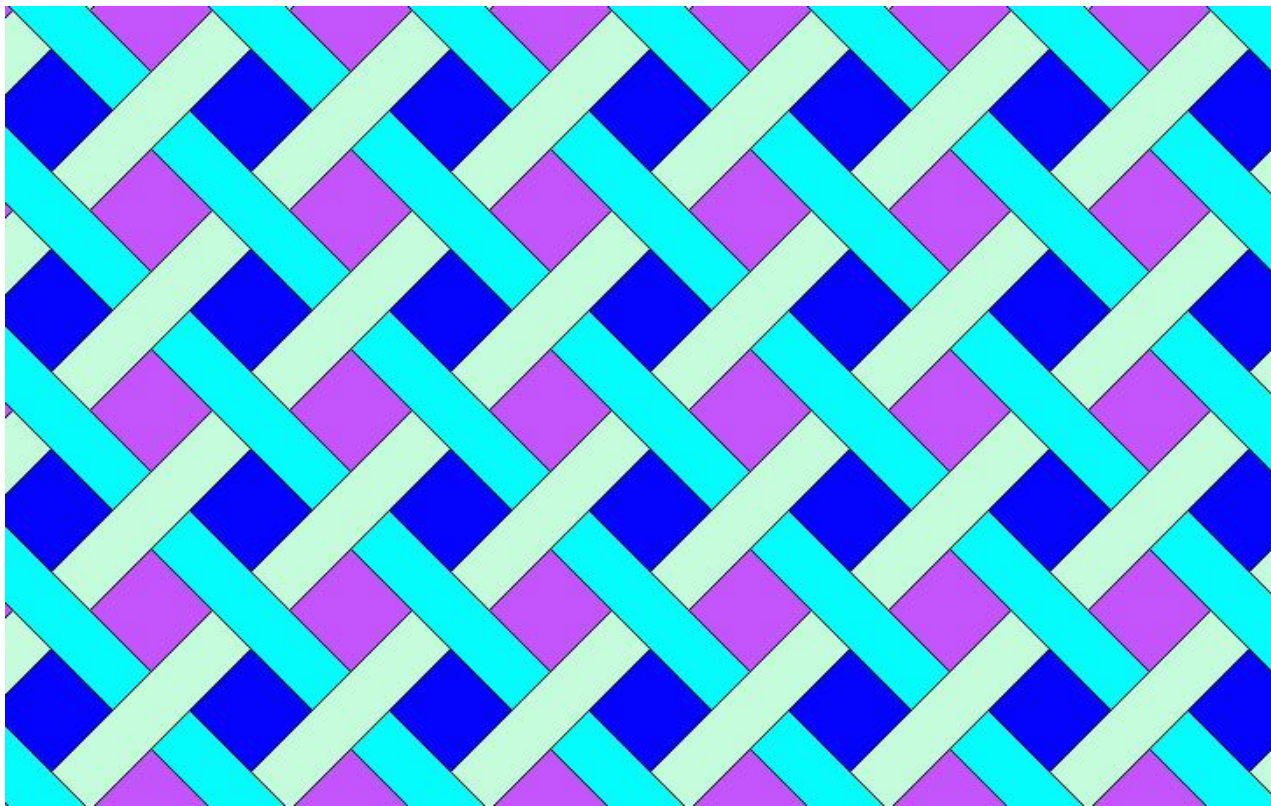
*442



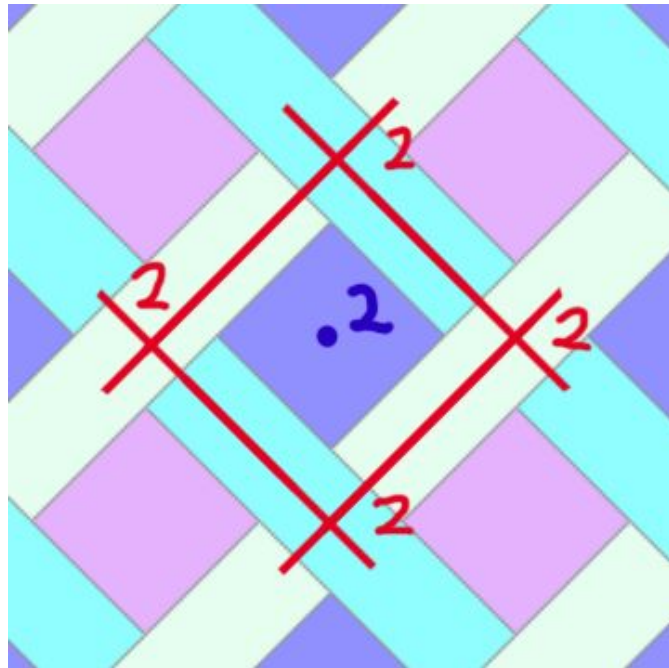
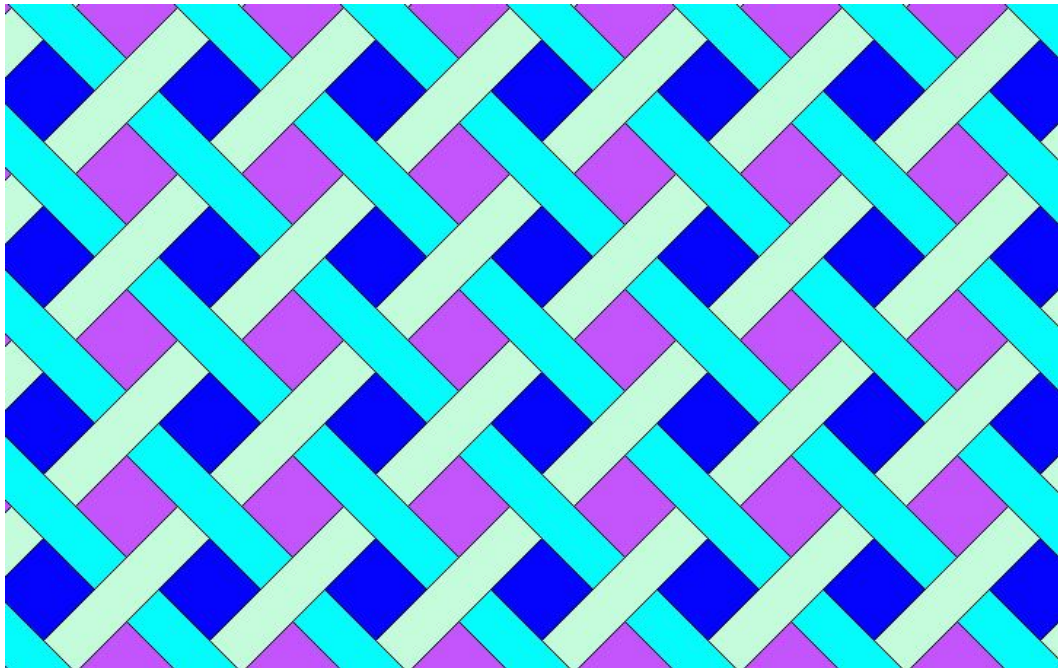


442



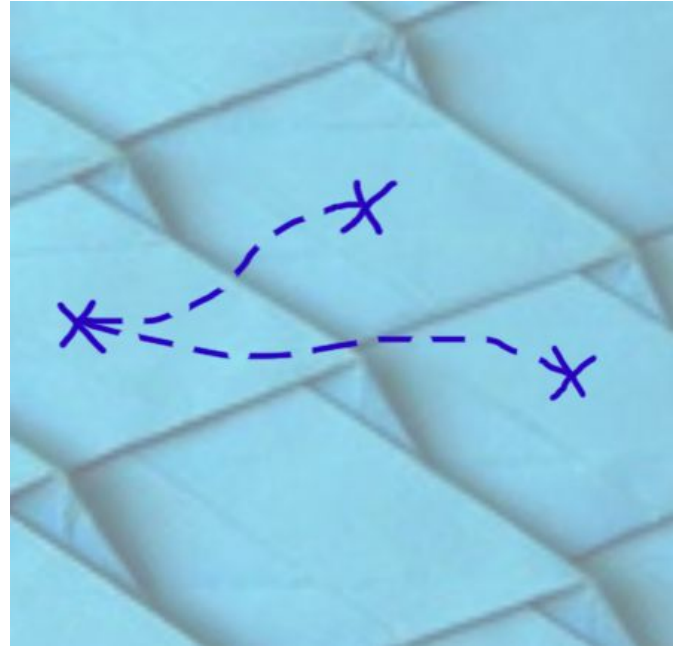


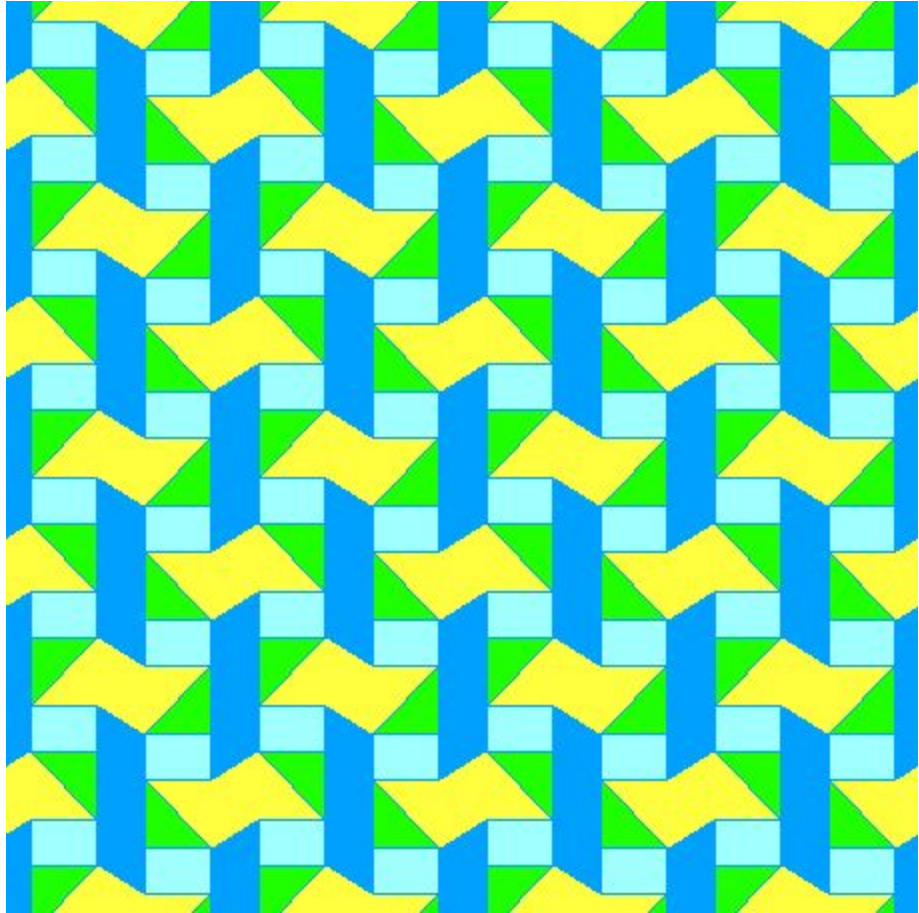
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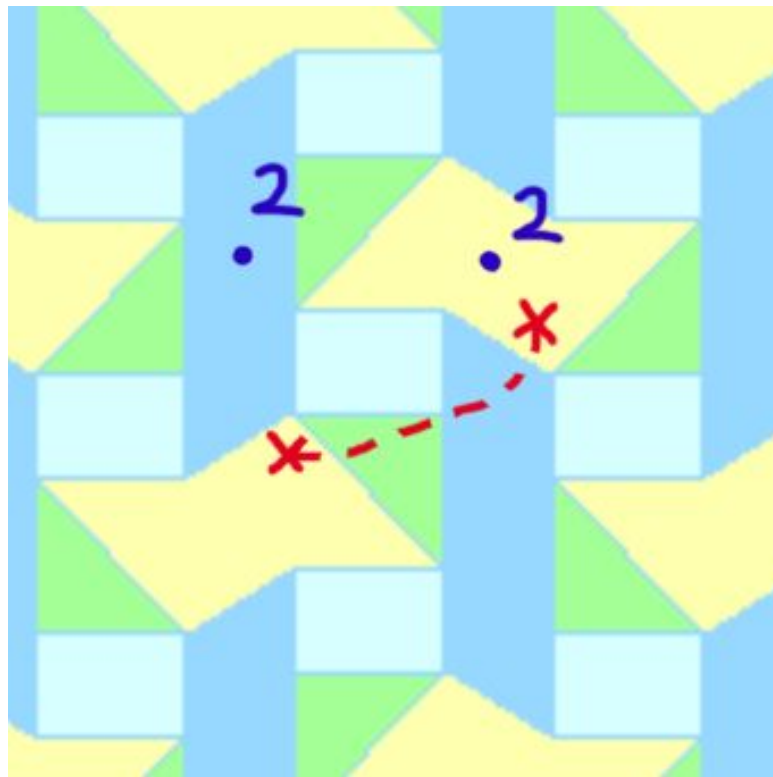
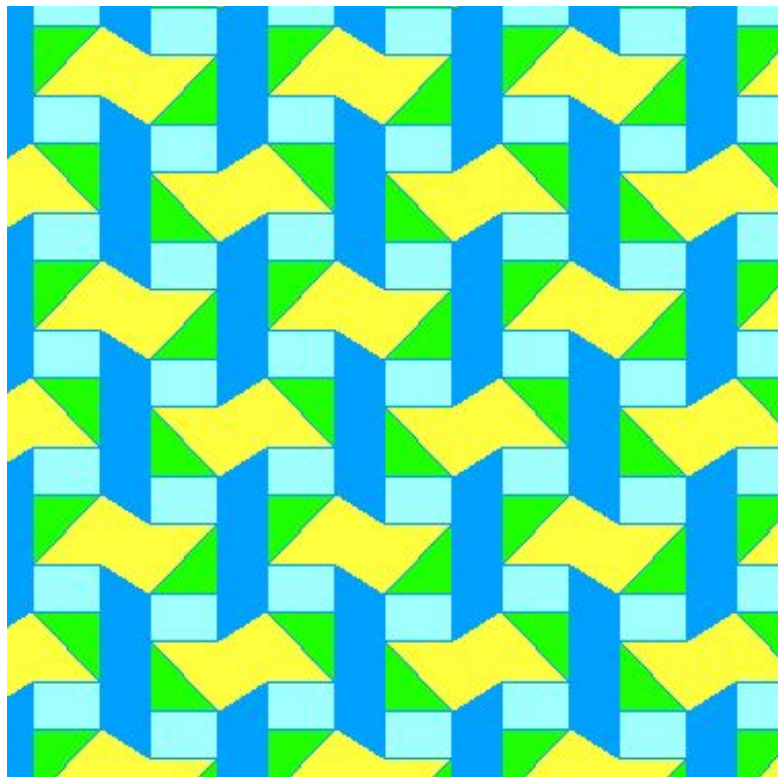


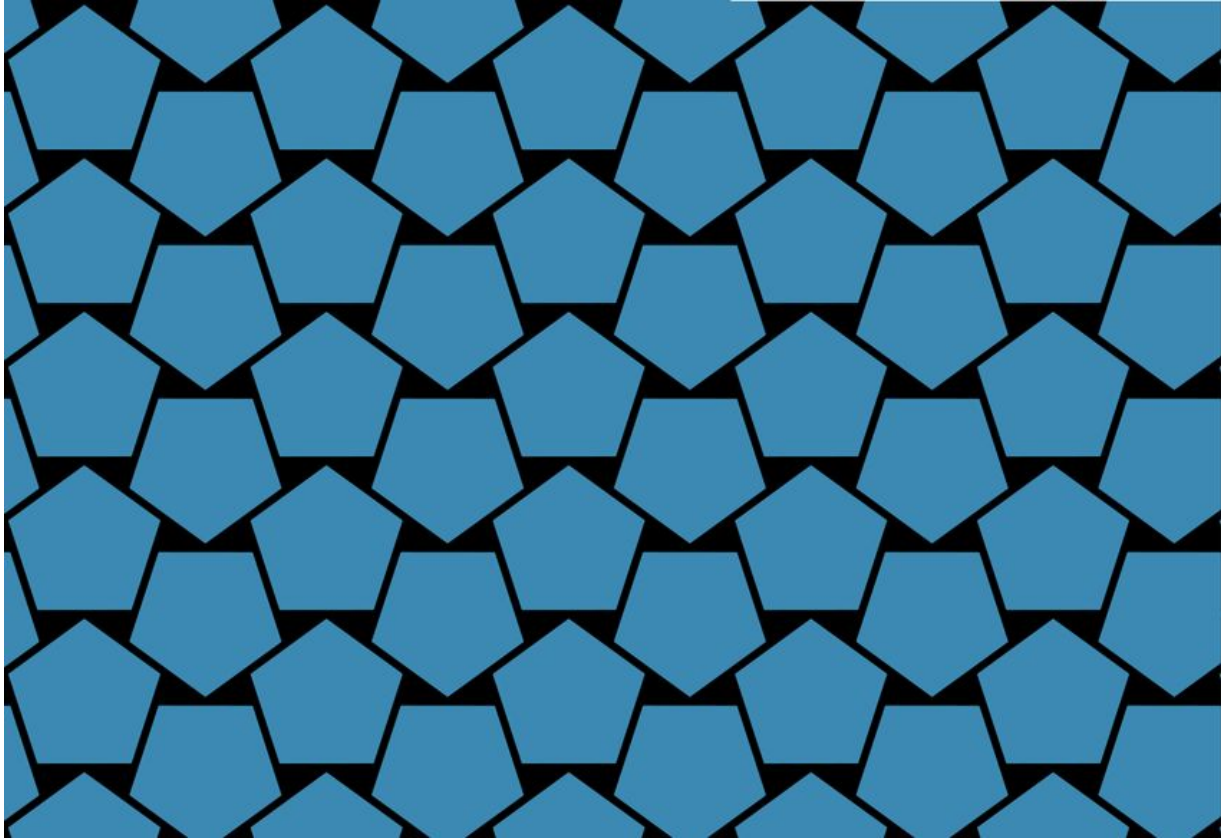
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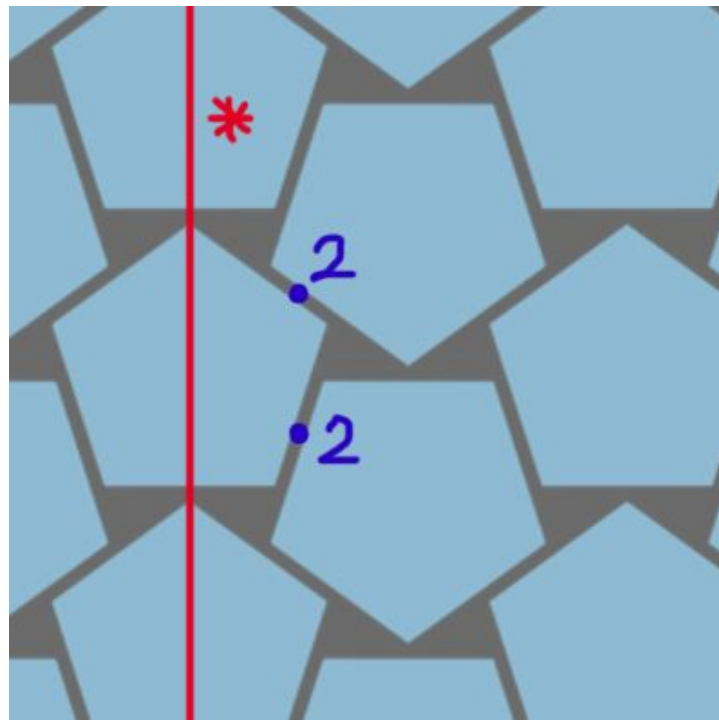
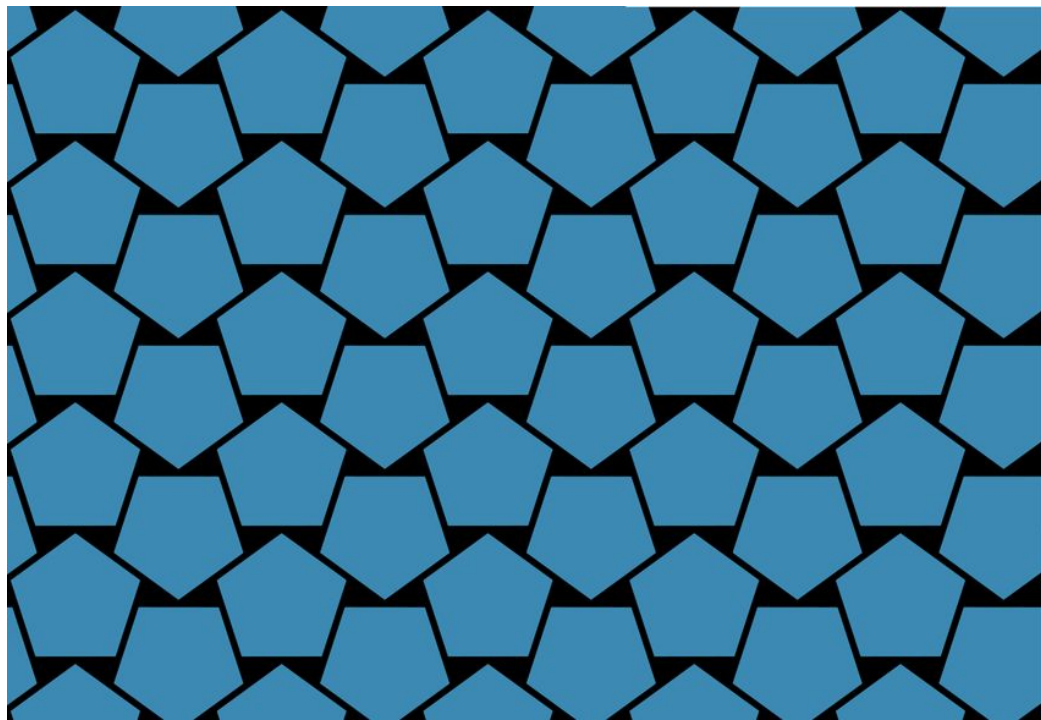


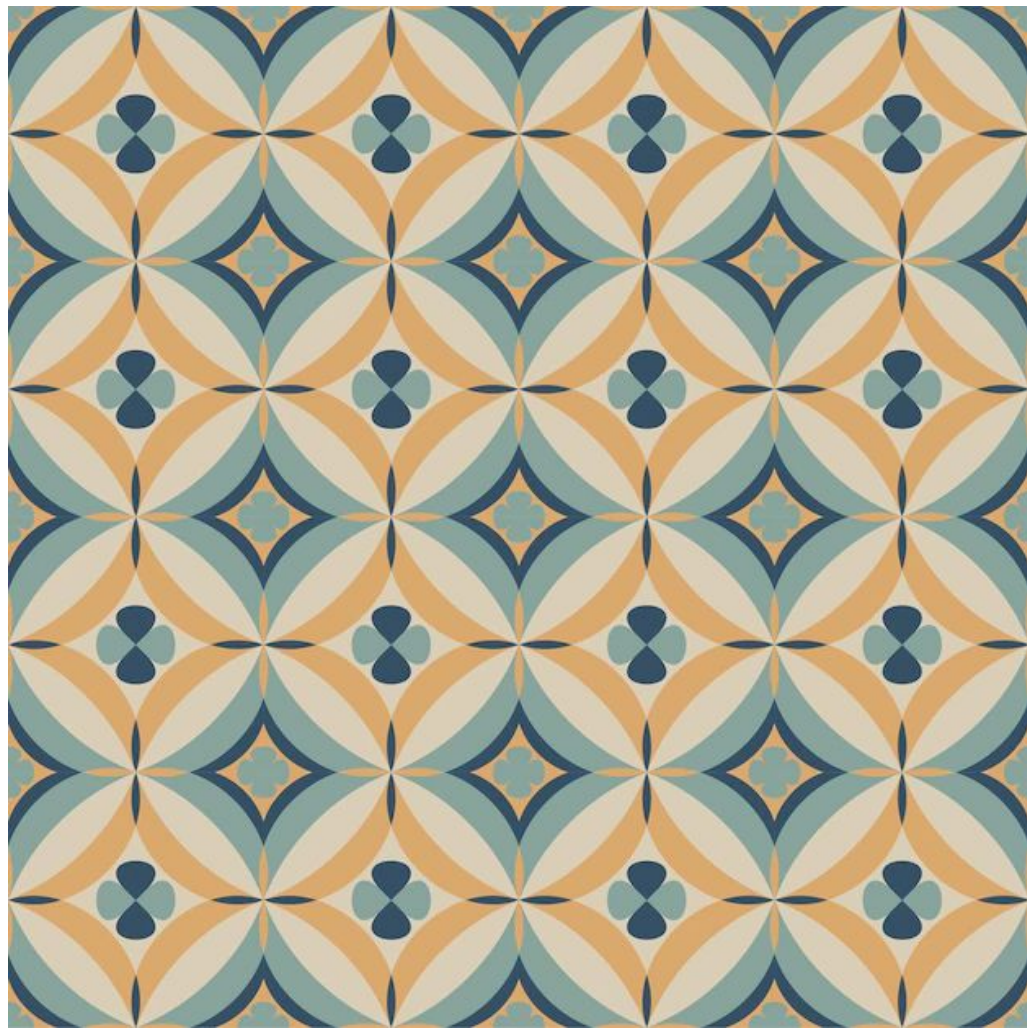
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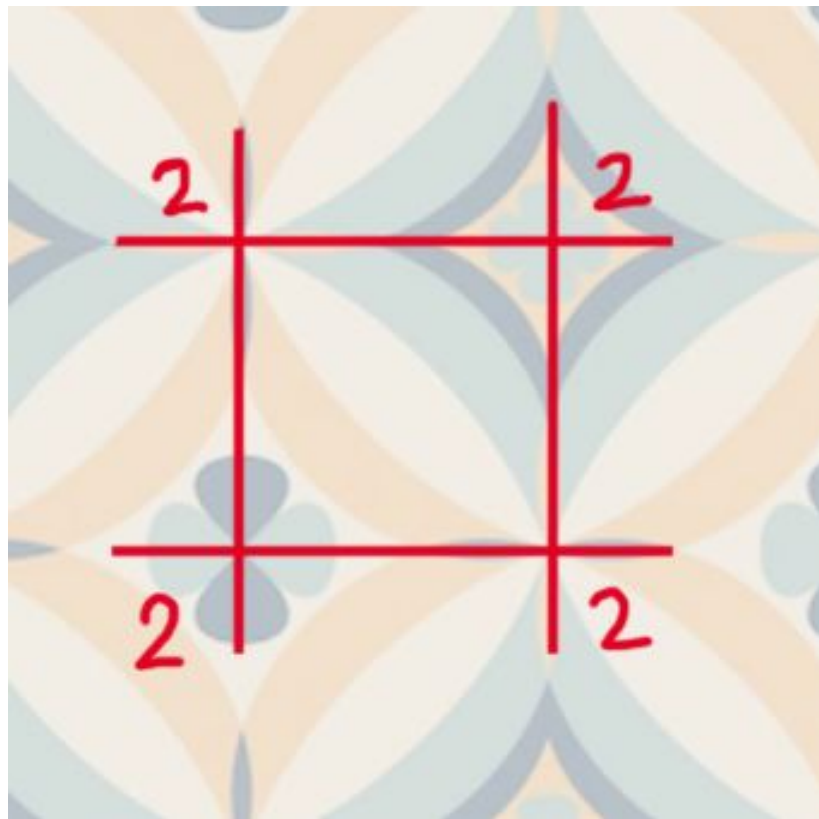
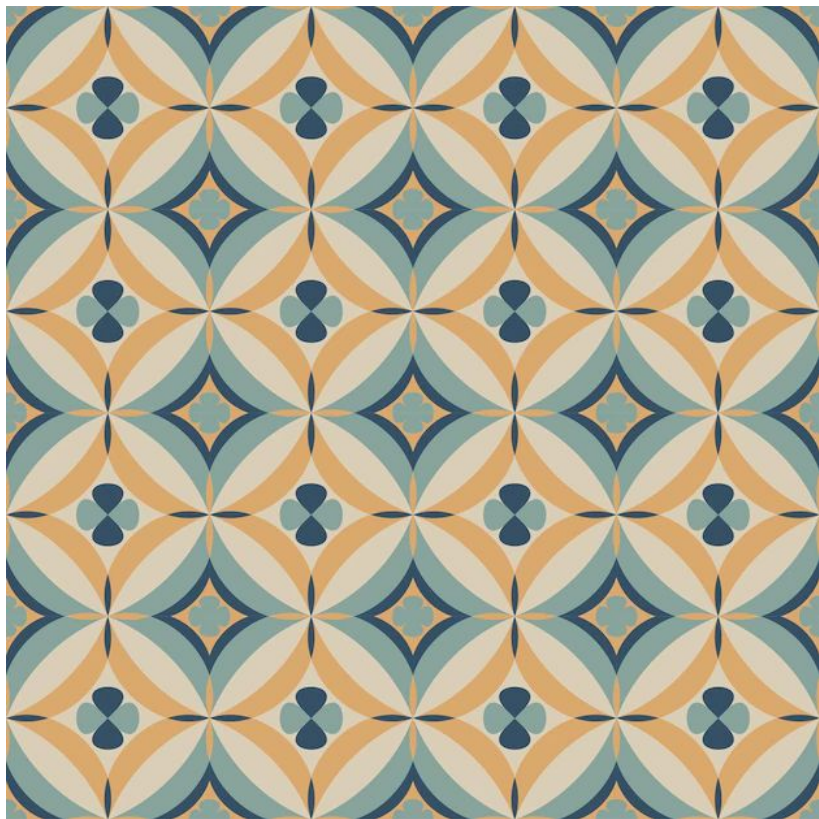


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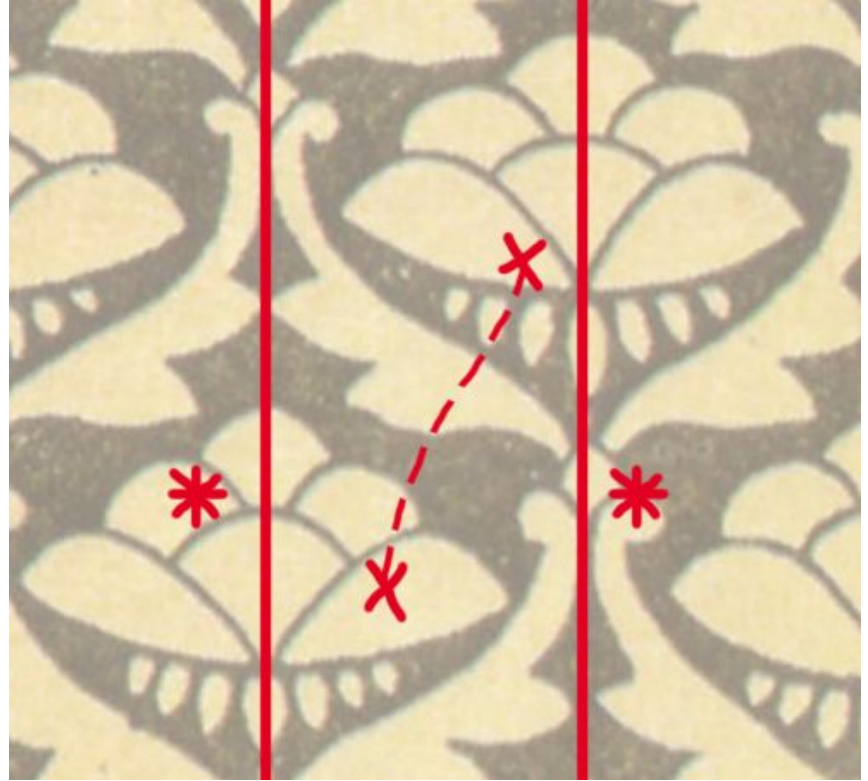


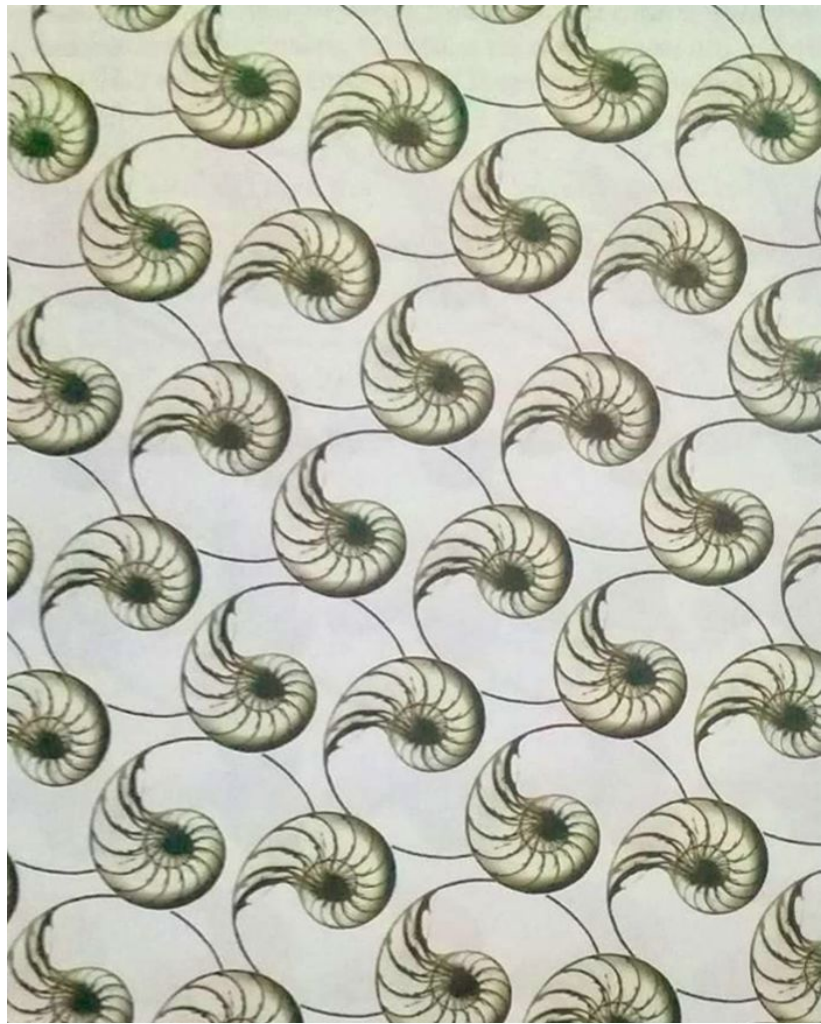
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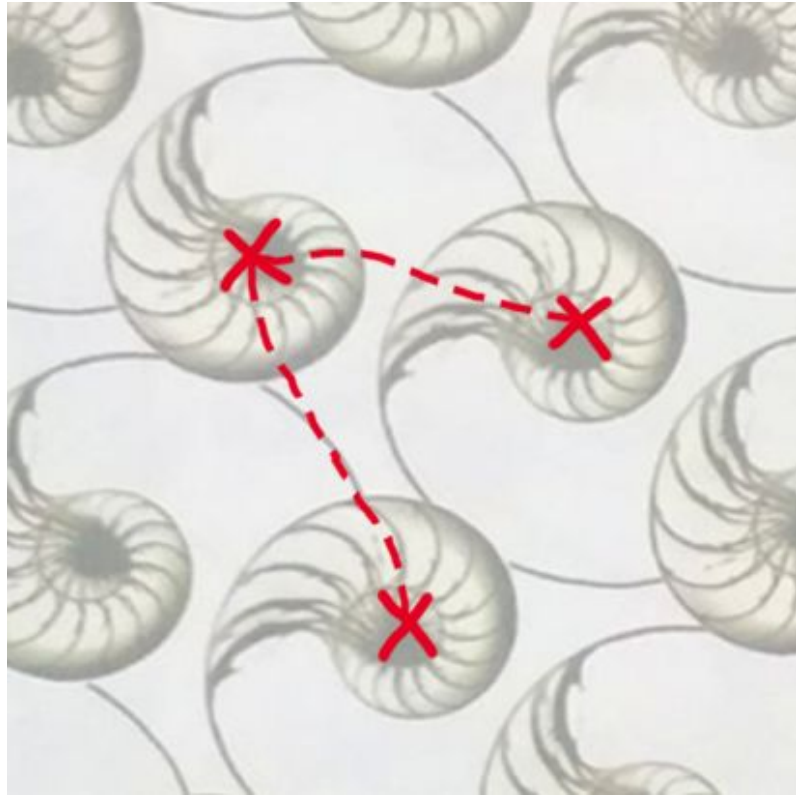


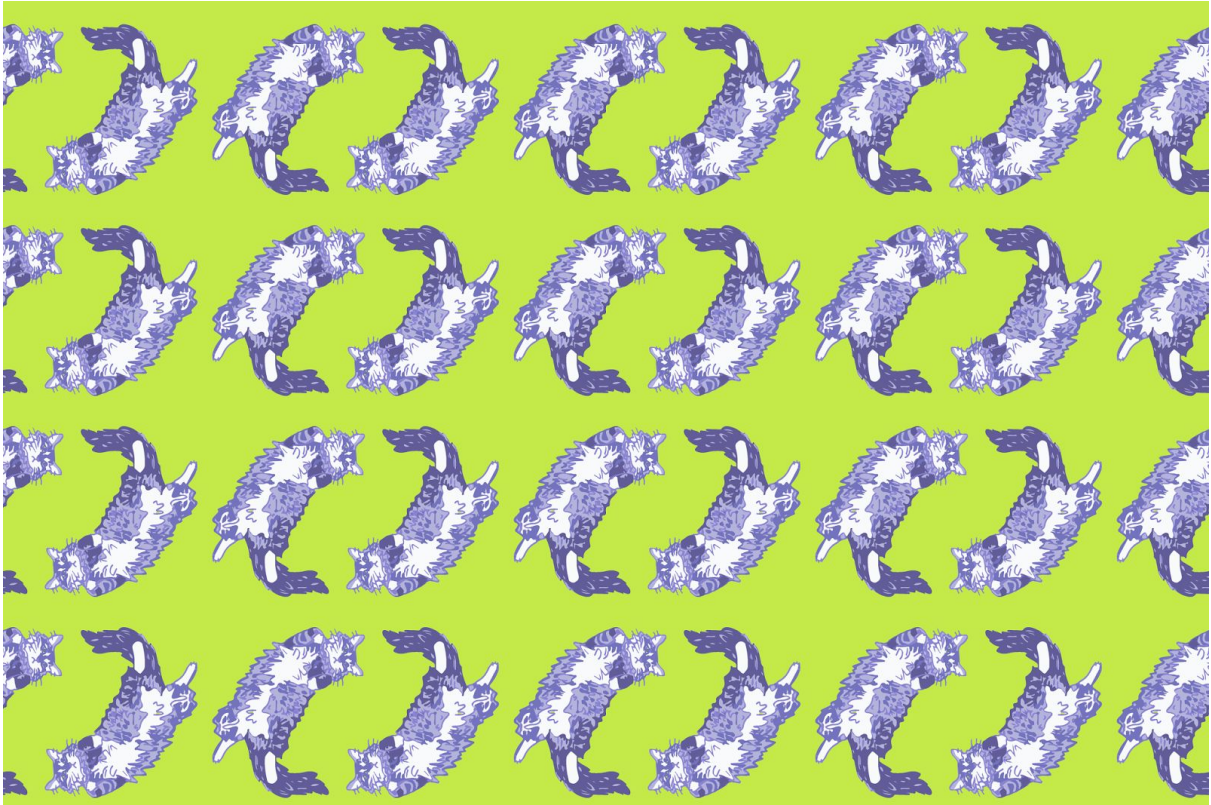
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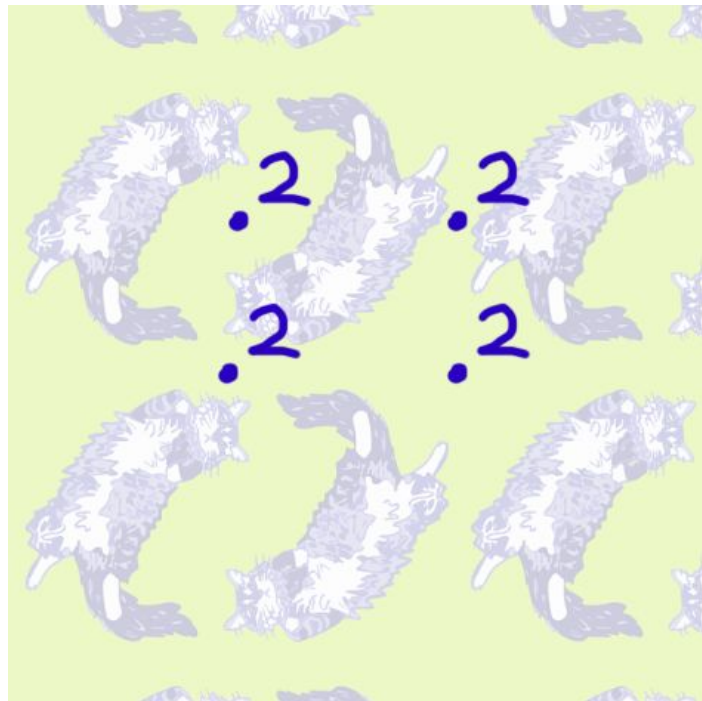
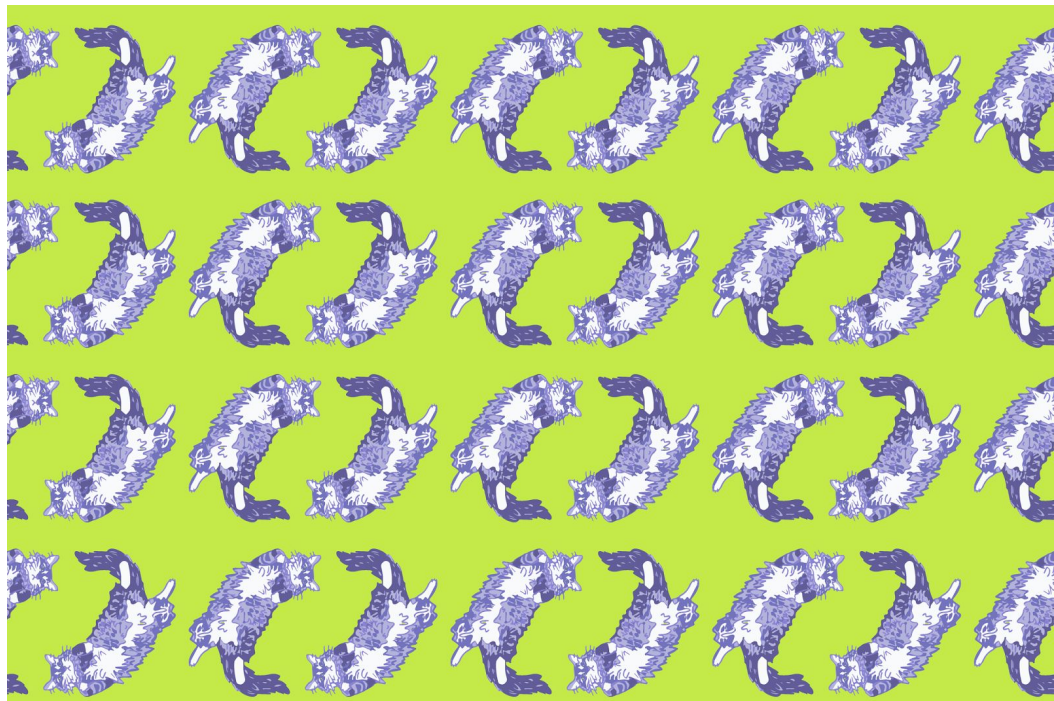


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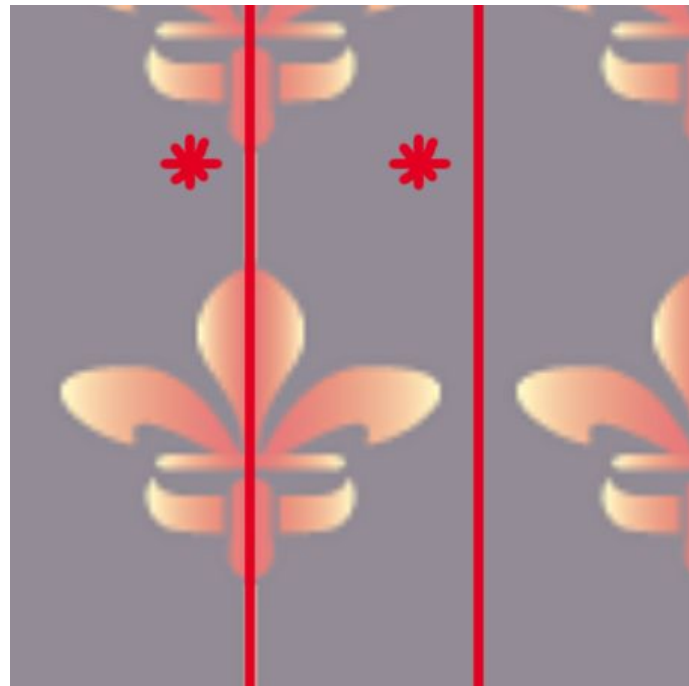


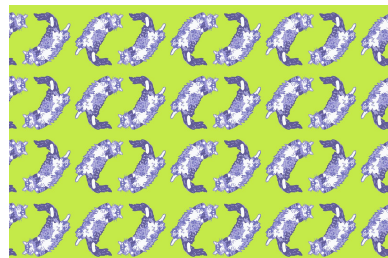
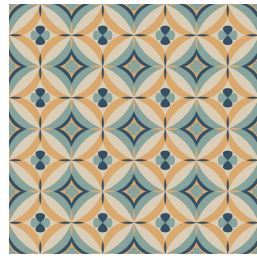
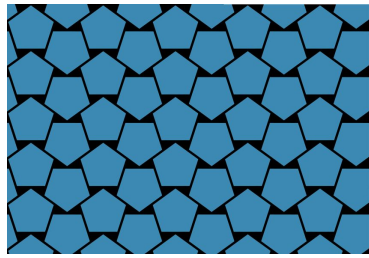
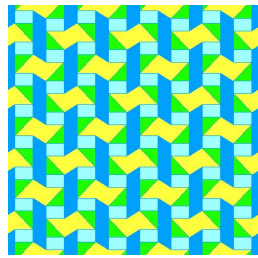
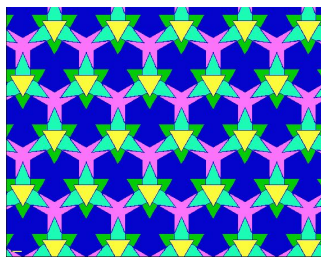
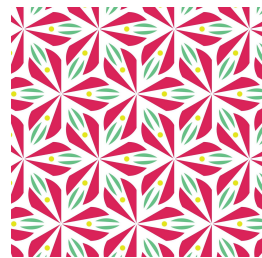
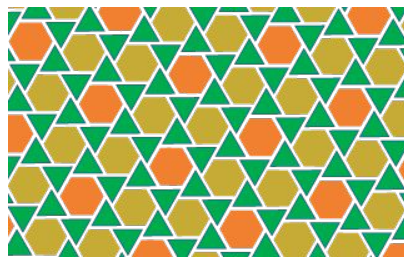
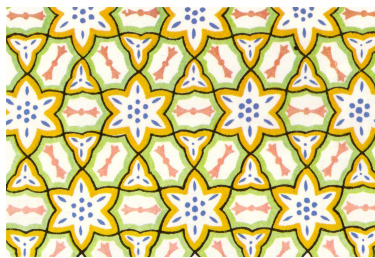
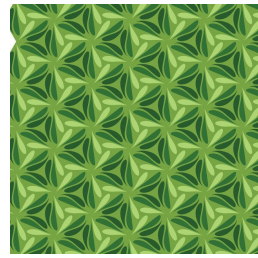
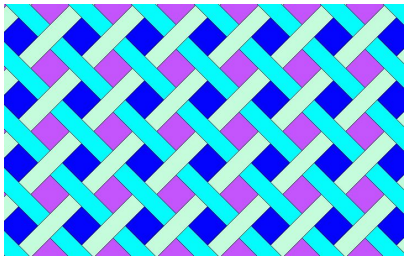
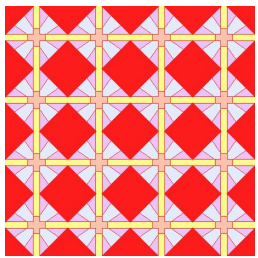
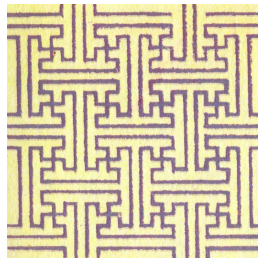
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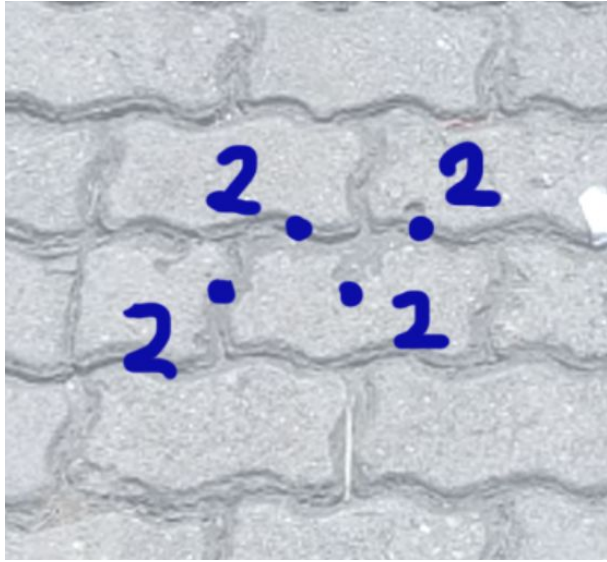


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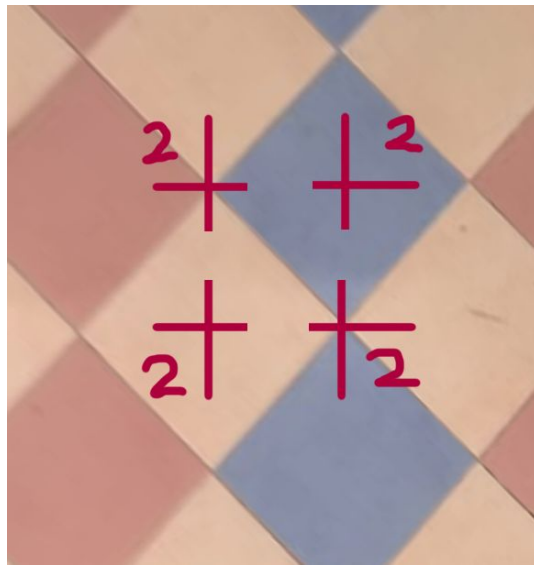




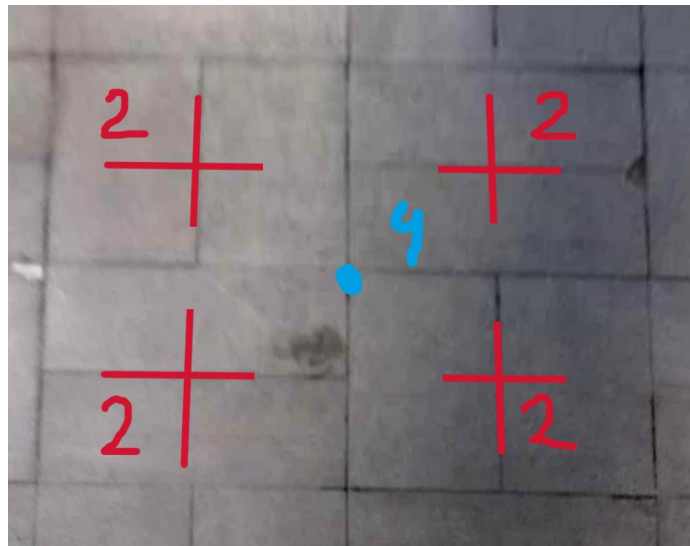
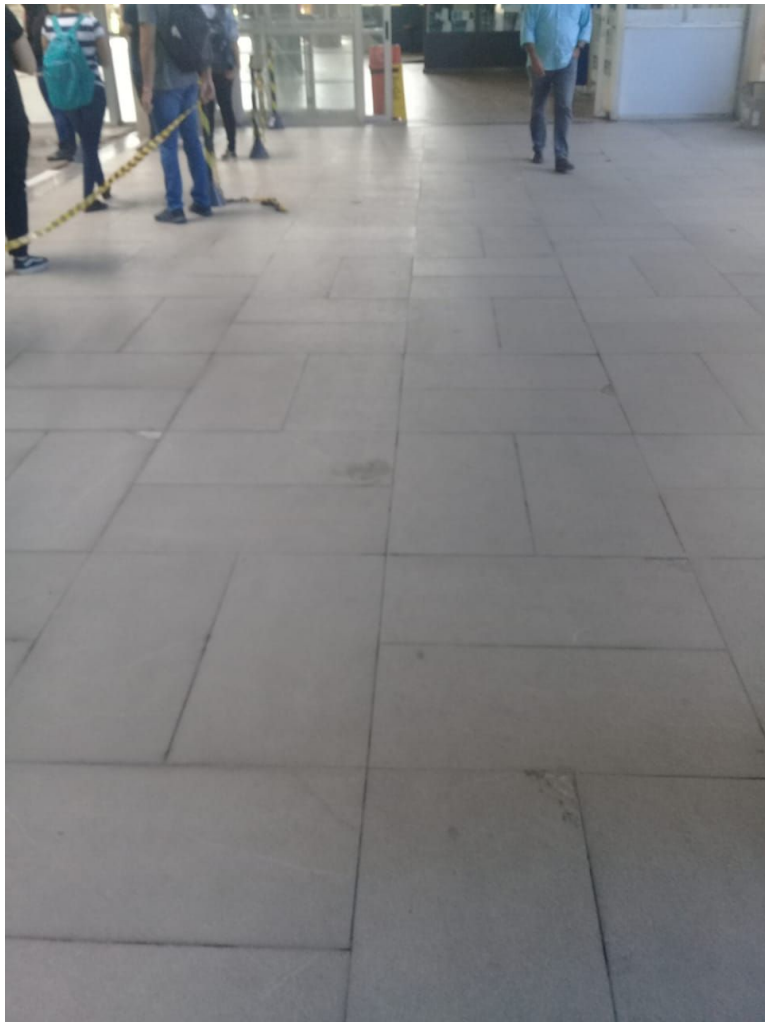
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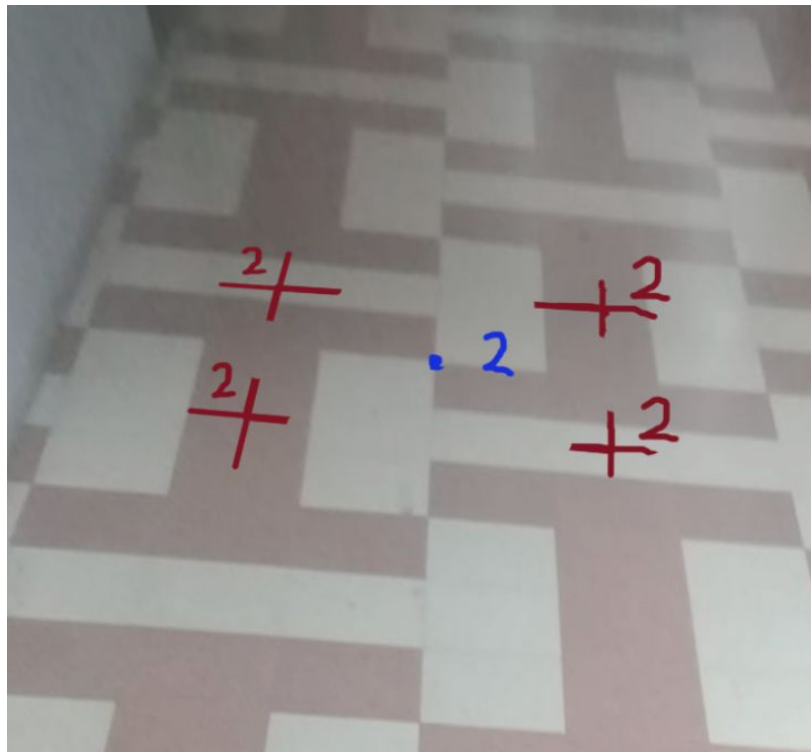
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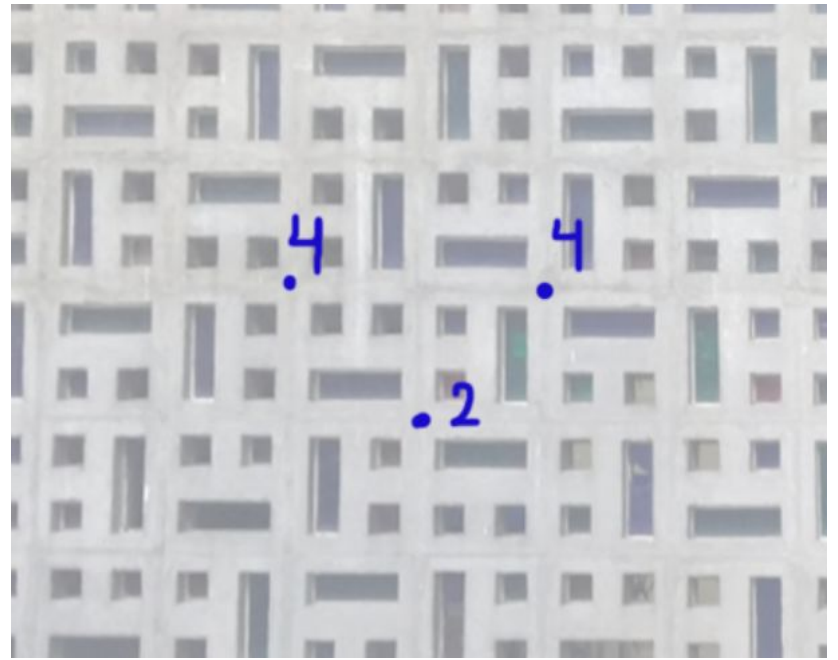
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$$4 * 2$$



$$2 * 22$$



442

442



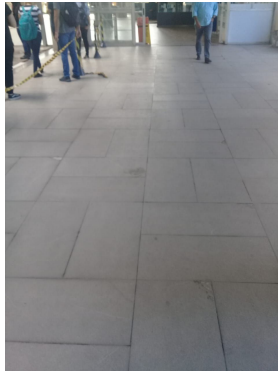
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*2222



4*2



2*22

